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Coupling between death spikes and birth troughs. Part 2: Comparative analysis of salient features

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HIGHLIGHTS

- Sudden death spikes produce birth troughs 9 months later.
- The relationship between spike and trough amplitudes is hyperbolic.
- This effect can also be identified on annual data.

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ABSTRACT

In part 1 we identified a coupling between death spikes and birth dips that occurs following catastrophic events such as influenza pandemics and earthquakes. Here we seek to characterize some of the key features of this effect. We introduce a transfer function defined as the amplitude of the birth trough (the output) divided by the amplitude of the death spike (the input). It has two salient features: (i) it is always smaller than one so is an attenuation factor and (ii) as a function of the amplitude of the death spike, it is a power law with exponent close to unity.

Since many countries do not publish monthly data, merely annual data, we attempt to extend the analysis to cover such data and how to identify the death–birth coupling. Finally, we compare the responses to unexpected death spikes and those to recurrent seasonal death peaks, such as winter death peaks.

1. Introduction

The case-studies described in the previous paper ([1], thereafter referred to as "Paper 1") specified some of the conditions which must be fulfilled for this effect to exist. The fact that it took place for the H1N1 crisis in Hong Kong but not for the attack of 9/11 in New York led to the idea that it is not really the number of deaths which is the main determinant, but rather the total number of persons who experience an adverse shock in their living conditions.

In the present paper we have three objectives.

(1) In Paper 1 the coupling was represented as an input–output effect (see Fig. 2a). It is therefore natural to measure how the transfer function of this system changes as a function of the magnitude of the initial death spike. In particular we wish to see if it is linear or nonlinear.

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(2) Secondly, we wish to extend the analysis of the coupling effect to cases for which only annual data are available. This would represent a significant extension for monthly data are unavailable in many developing countries, either because they are collected but not sent to the central government or because the central government gets them but does not publish them.

(3) Apart from the exceptional death spikes due to special events, monthly mortality data display also seasonal peaks. The amplitudes of such peaks are country-dependent and in some countries they reach levels which are as high or even higher than the exceptional death spikes. It is therefore natural to compare their respective effect on birth numbers.

The first of these objectives will provide a real predictive power. Based on the number of deaths of an event, the law of the Bertillon effect will allow us to predict the birth rate reduction nine months later. As another implication, once a model will be proposed its first requirement will of course be to be consistent with the hyperbolic Bertillon law.

2. Attenuation factor as a function of death spike amplitude

In Paper 1, it was suggested that the main determinant is the number of persons who experience an adverse shock in their living conditions. Unfortunately, in many cases this number is not well defined. For instance the measure of the incidence of a disease is highly dependent upon the criterion that is used:

- The number of persons hospitalized gives a low measure of incidence.
- A broader measure of incidence is through the number of working days lost to sickness in the labor market.

However, statistical data corresponding to these criteria are rather sparse and not comparable across a set of countries.

In the case of earthquakes we suggested that the shock on survivors could be measured through the number of "damaged houses" but this latter notion is itself a matter of appraisal.

For all these reasons the number of deaths remains the most convenient parameter for it has a clear significance and is widely available in vital statistical records.

2.1. Method

As for all time series which show a seasonal pattern we need to resolve how to handle it. The methods that we will use successively rely on two different conceptions of the phenomenon under consideration.

2.1.1. All inclusive conception (1)

In the first conception we consider the death spike as being of the same nature as the seasonal fluctuations. In other words it is seen as a seasonal fluctuations which just happens to be somewhat higher than the others. In this conception it would not make sense to separate the two effects. This means that we measure the amplitude of the death spike (and similarly for the birth trough) just "as it is". The beginning of the spike will be defined as the month where the number of deaths starts to increase after having been decreasing or flat. Similarly, the end of the spike will be the month where the deaths start to level off or to increase. Naturally, even if there is a small local dip in the upward phase or a local surge in the downward phase we do not wish them to be taken into account. That is why we perform a 3-point centered moving average before implementing the previous procedure.

2.1.2. Seasonal fluctuations seen as noise (2)

In the second conception in which one considers that the death spike is of a different nature than the seasonal fluctuations, the challenge is to remove the seasonal variations in the "best" possible way. In principle, the way to do that seems fairly evident and consists in dividing the monthly deaths of year y_0 by the seasonal profile that we denote by P_s (it is a set of 12 numbers). But how should the seasonal profile be defined? The answer depends upon the characteristics of the seasonal pattern. The simplest way is to take the monthly death profile of the year y_{-1} preceding y_0, in other words: $P_s = D(y_{-1})$. The main advantage of such a choice is the fact that in case there is a drift of the seasonal profile in the course of time, the year closest to y_0 will be the most appropriate.

A possible drawback of taking $D(y_{-1})$ is the fact that, as a single year, it may differ from the average seasonal pattern. Instead of taking only one year it is tempting to think that an average over several years would better approximate P_s. Is that true?

If the inter-annual statistical fluctuations of seasonal variations are small, then the average of n years will indeed converge toward a reasonable seasonal pattern. However, one should observe that in such a case $D(y_{-1})$ differs little from the average and is also a good choice therefore.

On the contrary, if from year to year there are large random changes in the monthly pattern, then an average of several years will be almost flat and the more years one takes the flatter it will become.[1] Such an average will be useless therefore and in such a case $D(y_{-1})$ will probably be the best choice as being close to y_0.

In summary we retain two procedures:

(1) Scaling the spikes and troughs just "as they are".

(2) Scaling them after dividing them by $D(y_{-1})$ and $B(y_{-1})$ respectively.

In what follows we will try successively the two procedures.

1 In order to discuss this point theoretically one would have to know the statistical frequency functions of the deaths (or births) in each months and also the interdependence of deaths in neighboring months. The statement that the average of several years tends to become level relies on tests performed on Japanese death data.

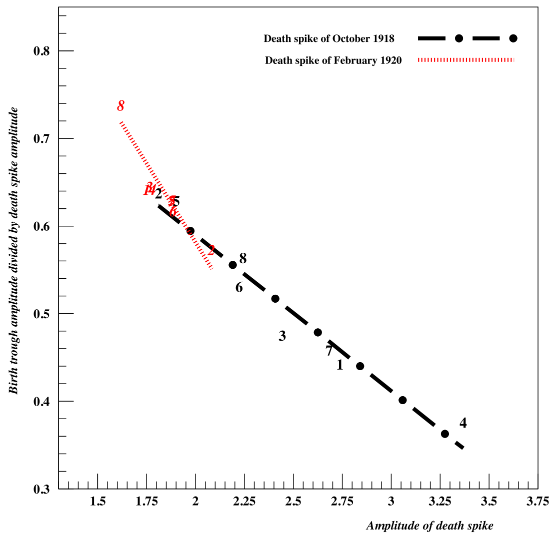

Fig. 1. Relationship between the influenza death spikes of 1918 and 1920 in the United States and subsequent birth troughs. The graph describes the attenuation factor: $R = A_b/A_d = f(A_d)$ where: A_d = amplitude of death spike, A_b = amplitude of birth trough. Both amplitude measurements were based on monthly data. As the death spike of 1920 was markedly smaller than the one of 1918 it permits an exploration of the small A_d section; this exploration suggests that the function $R = f(A_d)$ is probably nonlinear. The meaning of the numbers is as follows: 1 = Massachusetts, 2 = Michigan, 3 = New York, 4 = Pennsylvania, 5 = Indiana, 6 = Ohio, 7 = Cities of the Registration Area, 8 = Rural parts of the Registration Area. The regressions read as follows (the confidence intervals are for a confidence level of 0.95): 1918: $R = aA_d + b,\ a = -0.18 \pm 0.04,\ b = 0.94 \pm 0.02$ (correlation $= -0.97$). 1920: $R = aA_d + b,\ a = -0.36 \pm 0.15,\ b = 1.3 \pm 0.02$ (correlation $= -0.88$).
Source: Bureau of the Census: Mortality Statistics 1917–1921 [3]; Bureau of the Census: Birth Statistics 1917–1921 [2].

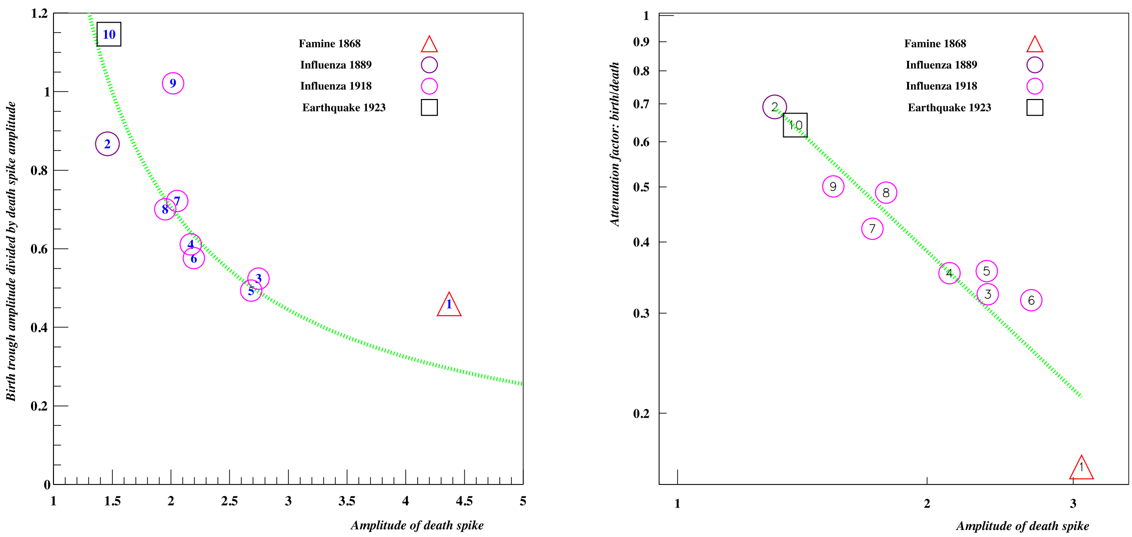

Fig. 2. Relationship between the amplitude of death spikes and the attenuation factor birth/death. Both graphs represent the ratio $R = A_b/A_d$ where: A_d = amplitude of death spike, A_b = amplitude of birth trough. The graphs differs by the method used in the calculation. Both results are compatible with a power law relationship. In Fig. 2b this relationship is shown in log–log scales. The term ''attenuation factor'' expresses the fact that the ratio R is smaller than one; the point higher than 1 in Fig. 2a may be a statistical fluctuation due to the fact that the smaller the amplitudes the more fluctuating their estimates. The two graphs rely on different conceptions of the fluctuations that are explained in the text. Overall, they lead to similar results, namely: $R \sim 1/A_d^{\alpha}$ where α is of the order of 1. The linear regression estimates for the logarithms, read $\log(R) = \alpha \log(A_d) + b$ with the following estimates for the parameters (confidence level is 0.95). Fig. 2a: $\alpha = -0.81 \pm 0.4,\ b = 0.25 \pm 0.11$ (correlation is -0.83) ; Fig. 2b: $\alpha = -1.37 \pm 0.4,\ b = -0.001 \pm 0.1$ (correlation is -0.94). The two estimates of α are compatible with $\alpha \simeq 1$. The plotted numbers correspond to the following cases: 1: Finland 1868 (famine); 2: France 1889 (influenza); 3–9: influenza epidemics of 1918 in several countries which did not take part in World War I: Sweden (3), Switzerland (4), Spain (5), Denmark (6), Finland (7), Chile (8), Japan (9); 10: Tokyo 1923 (earthquake and fire).
Source: Bunle [4], Finland [5]. Statistique Générale de la France [6], Nouvelle série (various years).

2.2. Selection of the data samples

Here, again, there are two different strategies.

(i) One can select an homogeneous sample of cases, such as the 1918 influenza epidemic in the US. This has the advantage of good comparability but the drawback is a fairly narrow interval for the amplitudes A_d of the death spikes.

(ii) One can consider a broad set of cases which includes famines, diseases, earthquakes, terrorist attacks. This has the advantage of a wider range for the death spike amplitude but has the drawback of increasing the noise by mixing different kinds of cases.

In what follows we will try both strategies. In strategy (i) the range of A_d will be $(1.5, 3.3)$ whereas in strategy (ii) it will be extended to $(1.4, 4.5)$.

2.3. The influenza pandemic in the United States

In the United States the development of the statistical network was slower than in smaller and more centralized countries like France or Sweden. Death statistics were recorded in the so-called Death Registration Area whereas birth data were recorded in the Birth Registration Area. In 1917 there were only 19 states in the Death Registration Area. However, some of them did not belong to the Birth Registration Area. That was for instance the case of California; as we need both death and birth data, California could not be used in our investigation. In addition we omitted a number of small states such as Delaware, New Hampshire or Rhode Island because for such states the monthly death numbers would be too low and therefore would show large fluctuations. That is why our sample comprises only 8 cases.

In the US, the war 1917–1918 has had little influence on married couples because husbands belonged to class IV of the "Selective Service System" which means that they were drafted only after the resources of the classes I, II, III had been exhausted. In short, one can admit that only a small percentage of husbands were drafted. Naturally, as can be expected, this rule incited many young people to get married to avoid the draft. From 1916 to 1921, according to Bunle ([4], p. 257) the numbers of marriages in Massachusetts were as follows (in thousands):

1916	1917	1918	1919	1920	1921
34.3	37.9	29.1	34.3	38.0	33.5

The data confirm that there was a marriage surge in 1917 and this effect is further confirmed by the monthly data. The United States declared war on Germany on 6 April 1917; in the 3 months Jan–Mar the marriages were almost the same in 1917 as in 1916 but in the quarter Apr–Jun they were 30% higher.

Fig. 1 shows that the level of noise is sufficiently small for a well-defined relationship to exist between A_d and the attenuation ratio $R = A_b/A_d$. The points 7 and 8 refer to urban and rural parts respectively and the fact that they are in line with the other points shows that the urban/rural factor does hardly affect the $R = f(A_d)$ relationship.

Not surprisingly, the level of noise is somewhat larger for the smaller death spikes of 1920 than for the spikes of 1918.

2.4. Broad sample of various death spikes

The small level of noise experienced in the previous data set encourages us to try a broader one. Fig. 2a shows a higher dispersion of the data points especially for the smallest death spikes but the level of noise remains acceptable.

Figs. 1 and 2a were made with methodology (1), whereas Fig. 2b was made with methodology (2); it can be seen that it is the case of Finland which is most affected but overall the two methods lead to similar results.

2.5. Nonlinearity of $R = f(A_d)$

Fig. 2a and b show clearly that, as already suspected in Fig. 1, the relationship $R = f(A_d)$ is not linear. Big death spikes have a lower attenuation factor than small ones but it seems to converge toward a limit.

A simple interpretation can be given. In paper 1 we have seen that in the earthquake of 2011 in Japan, some 400,000 houses were damaged. If we take this number as representing the persons directly affected the ratio to the number of deaths will be $M = 400{,}000/18{,}000 = 22$. Now, in the case of Finland in 1868, the number of excess deaths was 80,000. By applying the same multiplier $M = 22$ we get a number of $80{,}000 \times 22 = 1.8$ million. However, this number is equal to the whole population of Finland in 1868 which is unrealistic because wealthy persons were certainly not affected by the famine. In short, the large multipliers which are possible for small death spikes are not possible for large death spikes simply because of the limit imposed by the number of persons exposed to the risk.

3. Can one use annual instead of monthly birth–death data?

The investigation of the death–birth coupling requires high frequency (monthly or weekly) data; however when such data are unavailable the coupling can, under appropriate conditions, be identified through its specific signature at annual level. This is the point which will be discussed in this section.

Table 1
Monthly and annual fluctuations of birth numbers in Sweden (1821–1920).
Source: Bunle [4], Flora et al. [7].

	1 Coeff. of variation	2 Average of abs. changes	3 Stand. dev. of logs
Monthly births	6.8%	6.5%	6.8%
Annual births	4.2%	3.8%	4.2%
Ratio monthly/annual	1.60	1.70	1.61

Notes: The coefficient of variation is the ratio: standard deviation/mean. The second column gives the average of the absolute values of successive relative changes. The third column gives the standard deviation of the logarithms of birth numbers. The fact that the ratio monthly/annual is equal to 1.60 instead of $\sqrt{12} \simeq 3.5$ is due to the autocorrelation of the monthly birth data (see text). The data are for Sweden; the monthly data cover the 10 years Jan 1911–Dec 1920 (divided into 5 series of 2 years) while the annual data cover the 100 years 1821–1920 (divided into 10 series of 10 years).

3.1. Motivation

Although the Statistics Division of the United Nations publishes monthly birth and death data for many countries, there are quite a few important countries (e.g. China, India, Indonesia, Thailand) for which such statistics are not available. Even for countries included in the list the data are missing for some years. This raises the question of whether the pattern visible at the monthly level also results in a recognizable signature at annual level. If so, that would allow us to extend our analysis to a number of cases for which no monthly data are available; examples are the Tangshan earthquake in northeastern China on 28 July 1976, 4am (about 250,000 deaths), the Indian Ocean tsunami of 26 December 2004, 8am (250,000 deaths) the Great Sichuan earthquake in west China on 12 May 2008, 2:30pm (90,000 deaths).

The fact that for annual data there are no seasonal fluctuations should be a favorable factor but we first need to compare monthly and annual fluctuations of birth numbers.

3.2. Monthly versus annual fluctuations of birth numbers

Estimates of the fluctuations are given in Table 1. The data are for Sweden but are certainly similar for other countries. Instead of the rates we considered the numbers of births because in the early 19th century the total population was probably known with less accuracy than the birth numbers. Moreover, in order to avoid the bias due to the downward trend (related to the demographic transition) the global series were split into 10 annual series and 5 monthly series.

The three estimates considered in Table 1 are related but are not equivalent. Although not a standard one, estimate 2 is the most transparent for the present purpose.

As each annual value is the sum of 12 monthly numbers one would expect a coefficient of variation which is smaller by a factor $\sqrt{12} = 3.46$. Why then does it turn out to be only 1.60 times smaller? It is related to the fact that successive changes are not independent. The standard deviation $\sigma(n)$ of the average of n random variables of standard deviation σ and whose pair-wise correlation is on average equal to r is given by the formula[2]:

$$\sigma(n) = \frac{\sigma}{\sqrt{n}} g, \quad g = \sqrt{1 + (n-1)r}$$

For independent variables one gets the standard result: $\sigma(n)/\sigma = 1/\sqrt{n}$. Here, with $n = 12$ and $g = [\sigma(n)/\sigma]\sqrt{n} \simeq 3.46/1.6 \simeq 2.1$ one gets $r \simeq 0.33$.

Is this prediction consistent with the values given by the autocorrelation function ρ_j where j is the time lag expressed in months?

- A rough test is to observe that ρ_6 is of the order of 0.3.
- For a more accurate test one needs to compute the average of the correlations of all pairs of months. Naturally, the number of pairs depends upon the time-lag. For a time lag of 1 month there are 11 pairs $(1-2, 2-3, \ldots, 11-12)$, whereas for a time lag of 10 there are only 2 pairs $(1-11, 2-12)$. Altogether one gets: $r_p = (1/66)\Sigma_1^{11}(12-j)\rho_j$. Plugging in the values of the autocorrelations, one gets: $r_p = 0.36$ which is consistent with the value of r predicted above.

3.3. Conditions under which one expects a recognizable signature

At monthly level one observes the following succession of events:

The death spike is followed 9 months later by a birth trough which is itself followed 3 or 4 months later by a birth rebound (the rebound effect is documented in Paper 1). It is the predicted succession of these events which helps us to identify them. Under what circumstances can one expect a similar pattern for annual data?

- Consider a death spike which occurs in November of year y_0. The minimum of the birth trough would be expected in July of year $y_1 = y_0 + 1$. The rebound would start about 4 months later, that is to say in November of y_1; however, most of

2 The proof is straightforward and recalled in Roehner [8, p. 45].

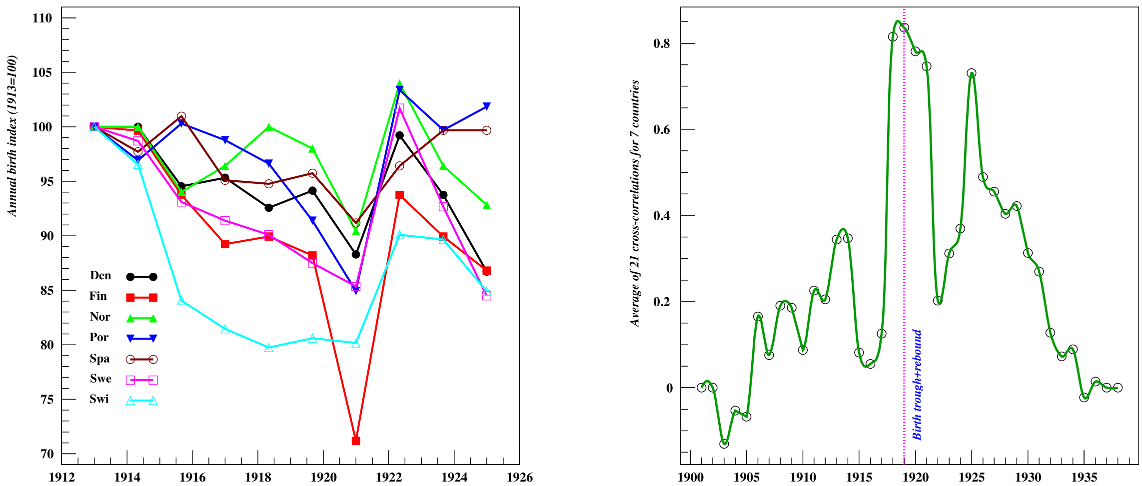

Fig. 3. Identification of systemic birth events within a set of countries. The graph on the right-hand side shows that the trough and rebound of 1919–1920 is the only major systemic birth event in the time interval 1900–1938. This is confirmed by the average cross-correlation shown in Fig. 3b. The graph 3b was made with a moving window with a width of 5 months.
Source: Mitchell [9].

it will occur in the following year, namely $y_2 = y_0 + 2$. This is the ideal case because it results in a well staged succession of yearly death and birth levels:

$$d(y_{-1}) < d(y_0) = \text{spike} > d(y_1) \quad b(y_0) > b(y_1) = \text{trough} < b(y_2) = \text{rebound} \tag{A}$$

Such cases will be referred to as "class A" cases. If the death spike occurs earlier in y_0 the situation will be less favorable.

• If the death spike occurs between January and April the birth trough will take place (partly or totally) in y_0 and the rebound will take place in y_1. This leads to the following signature which will be referred to as "class B".

$$b(y_{-1}) > b(y_0) = \text{trough} < b(y_1) = \text{rebound} > b(y_2) \tag{B}$$

• If the death spike occurs between May and October the birth trough will be between February and July of y_1. In this case whether $b(y_1)$ will be lower or higher than $b(y_0)$ will depend upon how fast the rebound starts and how strong it is. This mixed and fairly unclear situation will be referred to as "class AB".

In summary, one can remember the following rules. (i) If the death spike occurs in January or February one expects: $b(y_0) < b(y_1)$. (ii) When the death spike occurs in March or April one is in a mixed situation for which one does not expect any clear relationship for birth numbers. (iii) If the death spike occurs between May and December one expects: $b(y_0) > b(y_1) < b(y_2)$.

3.4. How to use annual data?

We now come to the most important part of this section. How can we apply what we have learned about annual birth data in order to make them into a useful tool? From the discussion above we know that we should select events which occurred toward the end of year. As the 1918 influenza pandemic in the northern hemisphere occurred in October–November it would be a good candidate. We know that the birth trough will be located sometime around July 1919 but with annual data it will be spread over the whole year and therefore become "diluted" about 6 times (if the monthly trough lasts two months). On the other hand the background noise will be reduced only by as factor 1.6. Thus the identification will be 3.7 times more difficult. This leads us to work in a statistical perspective that is to say by exploring a whole set of countries as done in Fig. 3b.

The set of countries consists of 7 European countries, none of which took part in the First World War. Fig. 3a shows that their birth fluctuations are fairly disconnected except for two changes which are common to most of them, namely the dip of 1919 and the rebound of 1920. This widespread accident appears as a correlation peak in Fig. 3b. This graph was made by computing the cross-correlations of the 21 pairs of countries over a moving window and then summing them up. The correlations add together destructively except in the interval around 1919–1920 and in a narrow interval around 1925.

In short, this method permits to identify collective motions of birth rates.

Can we repeat for the Indian Ocean Tsunami of 2004 the identification operation done in Fig. 3b? The answer is "no". The reason is simple. The 3 countries with the highest death rates were Sri Lanka (35,000 deaths, i.e. 1.7 per 1000), Indonesia (131,000 deaths, i.e. 0.65 per 1000) and Thailand (5400 deaths, i.e. 0.09 per 1000). So, there were only two countries with death rates over 0.1 per 1000. Moreover, for one of them, namely Indonesia, there are no annual birth and death data reported in the Demographic Yearbook of the United Nations.

Table 2
Characteristics of "normal" seasonal fluctuations of death and birth numbers.
Source: Bunle [4], website of the United Nations, Statistical Division.

Country	Period		Coeff. of variation	Correlation death–birth(tr-inv)
Japan	1906–1911			
		Death	10%	0.71
		Birth	22%	
Sweden	1906–1911			
		Death	12%	0.09
		Birth	4.4%	
Switzerland	1906–1911			
		Death	15%	0.77
		Birth	5.1%	
Japan	2004–2013			
		Death	9.3%	−0.39
		Birth	2.7%	
Sweden	2004–2013			
		Death	7.8%	0.73
		Birth	7.1%	
Switzerland	2004–2013			
		Death	8.6%	0.06
		Birth	3.6%	

Notes: The term "normal" in the title of the table means that no exceptional death spike occurred in the time intervals under consideration. The coefficient of variation is defined as the standard deviation divided by the average. In the definition of the correlation, "birth(tr-inv)" means that the birth data have been translated 9 months toward the past and inverted (i.e. replaced by their opposite) in conformity with the graphs drawn in paper 1. Here the error bars on CV are less than 20% of the results.

4. Exceptional versus seasonal death upsurges

Since exceptional death upsurges (as those considered so far) give rise to birth troughs, should one not expect similar responses for the recurrent upsurges of seasonal mortality? This is a question which comes about naturally and must therefore be addressed. However, we will see that it is not a well defined question in the sense that its answer is country dependent. This is due to the fact that, apart from the Bertillon effect, the fluctuations of birth numbers are also influenced by other factors, for instance climatic features as well as cultural and religious rules. As an illustration of the later, one can mention the study of Friger et al. [10] which analyzes the effect of the Ramadan month on births[3] and the more recent study of Herteliu et al. [11] which identifies a clear-cut reduction in the number of conceptions during the Orthodox Lent period in Romania. More broadly, the global effect of exogenous factors is clearly demonstrated by the following demographic features of Japan.

• In 1914–1917 the coefficient of variation (CV=standard deviation divided by average) is equal to 9.4% for the deaths and 29% for the births. As we have seen previously that the Bertillon effect is an attenuation (not an amplification) the fact that CV(birth) is three times CV(death) shows that there are other exogenous factors at work.

• The previous argument is comforted by the following observation. Between 1906 and 2013 CV(death) remained fairly constant around 10% whereas CV(birth) fell from 30% to 3.2%. This suggests a decline of the exogenous factors in the course of times.

The case of Japan would suggest that, in a general way, CV(birth) decreases strongly in the course of time. As a matter of fact such a conclusion would appear fairly natural for one may think that in former times sexual relations (and conceptions) were shaped by climatic conditions, cultural traditions and religious precepts much more strongly than they are nowadays. However, to our surprise, no matter how natural, this idea was not found consistent with observation. Switzerland offers a clear counter-example. In the time interval, 1878–1885 CV(birth) is as low as 3.6%. In subsequent years it increases to 9.0% in 1920–1923 and falls back to 4.1% in 2010–2013.

In conclusion to this discussion one can retain that the pattern of birth numbers is heavily influenced by exogenous factors which, in addition, appear to be strongly country-dependent.

In what follows we will try to answer a more limited question, namely is the birth response to a death peak of given magnitude the same no matter whether the later is unexpected or on the contrary a regular occurrence.

What analytical tool should be used to answer this question? The intercorrelation may be the first idea which comes to mind but it is not satisfactory for an obvious reason. We wish to single out the responses to death rate peaks whereas the linear correlation will also reflect the response to death troughs or to flat death rates. In addition, because of the attenuation, no visible birth coupling should be expected when the death peak is too small. Despite its limitation the correlation can give valuable information in two opposite cases:

• A correlation of 0.70 or higher cannot be obtained if the peaks do not coincide.

• A negative correlation will indicate that the peaks do not coincide.

[3] It appears that, for some unknown reason, there is a slight increase of the birth rate during Ramadan.

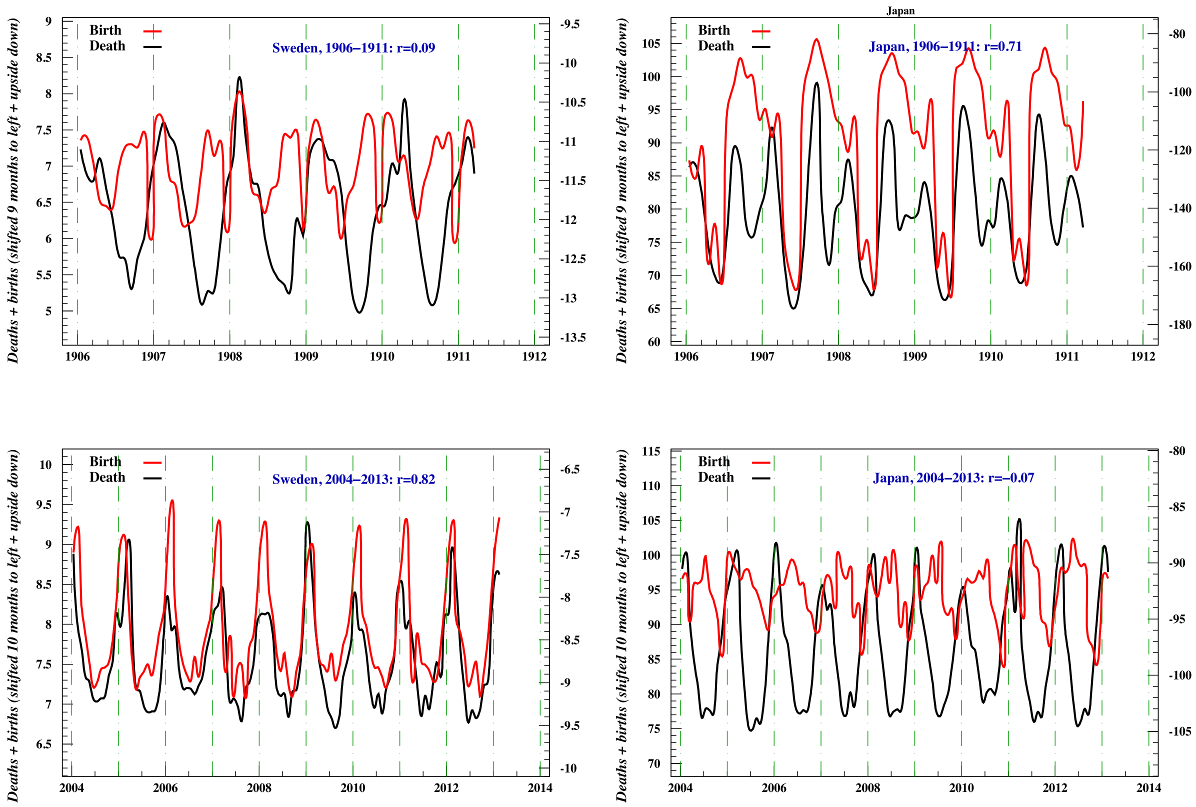

Fig. 4. Is there a correlation between seasonal death peaks and conception troughs in the early 20th and 21st centuries? **1906–1911**: The death curves are very different in the two countries: in Sweden there is a winter peak whereas in Japan there is a summer peak. The conception curves are also very different. For instance, in Sweden there is a sharp conception spike at the end of each year which may be due to Christmas time. **2004–2013**: Whereas the death peaks in Sweden and Japan are very similar, the conception curves are very different. In Sweden the peaks of the inverted conception curve coincide closely with the death spikes which results in a high correlation between the two series. On the contrary in Japan, the death and birth series are disconnected which results in a correlation close to zero. This shows that there is no systematic connection between deaths and births. The high synchronization observed in Sweden cannot be considered as the rule and is certainly due to special circumstances. This is confirmed by the fact that other cases (e.g. Switzerland) are intermediate between the extreme cases of Sweden and Japan.
Source: 1906–1911: Bunle [4]; 2004–2013: Website of the Statistical Division of the United Nations.

Naturally, this argument holds only under two conditions.

(i) The amplitude of the seasonal death fluctuations must no be too small for otherwise, even if the effect exists, it will be too small to be detected at birth level. In order to test this argument we use the methodology of extreme cases that is to say, we compare two cases, one in which the CV of the birth series is small and another in which it is large (say about 30%). In addition we require that the two cases occur approximately in the same time window in order to ensure similar environment conditions.

What conclusions can one draw from the results given in Table 2 and Fig. 4?

(i) The CV of the death series are fairly stable at a level of about 8% both in time and across countries.

(ii) The CV of the birth series are fairly different from country to country; thus, in 1906–1911 they range from 4.4% in Sweden to 22% in Japan. Contrary to our expectation based on the argument given in the text above, the CV do not, as a rule, decrease in the course of time. It is true that in Japan there is a considerable decrease but in Sweden the CV increases from 4.4% to 7.1%.

(iii) It is in the correlation that we are the most interested. How can one explain that in some cases the correlation is fairly high? Our explanation is that this occurs purely by chance. One can give a fairly crude argument. For present-time data one can safely assume that the death series has only one spike which occurs in winter time usually in January or February.[4] On average this peak has a width of about 3 months. If, as is the case for Sweden (2004–2013), the birth series has also only one peak then they may more or less overlap with probability $3/12 = 0.25$. On the contrary, if the birth series has a more complex structure, for instance with two peaks (and therefore two troughs), then the single death peak can be in sink only with one of the birth troughs which will result in a low correlation, as seen for Japan in 2004–2013.

4 In warm countries and former times there may also be a death peak in summer time due in particular to enteritis of babies and children.

5. Conclusion

When irregular, seasonal fluctuations can be a major source of noise but fortunately in many countries they are sufficiently regular to be treated as deterministic signals.[5] This is what permitted the detection and analysis of the coupling effect between death spikes and birth troughs.[6] We have shown that the birth/death ratio is always less than one and that, as a function of the amplitude of the death spikes, this attenuation ratio follows a hyperbolic power law.

As always when fairly accurate measurements become possible, they raise a number of questions. How can one explain that there is no coupling for 9/11 or for the winter death spikes in Japan? If one could get monthly birth and death data at province level for large countries like China, India or Indonesia that would certainly allow further progress.

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5 In contrast suicide rates have also a substantial seasonal component but it is much less regular than the birth and death signals considered here.

6 Despite the fact that Jacques Bertillon [12] analyzed only one instance, namely the influenza epidemic of 1889, he should be credited with this discovery; therefore the effect can be referred to as the "Bertillon effect".