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Predictive implications of Gompertz's law

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HIGHLIGHTS

- Global death rate data show human life is limited to 120 years.
- This allows prediction of the slope of Gompertz's law.
- Moreover maximum age proves independent of country-size.

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ABSTRACT

Gompertz's law tells us that for humans above the age of 35 the death rate increases exponentially with a doubling time of about 10 years. Here, we show that the same law continues to hold up to age 106. At that age the death rate is about 50%. Beyond 106 there is so far no convincing statistical evidence available because the number of survivors are too small even in large nations. However, assuming that Gompertz's law continues to hold beyond 106, we conclude that the mortality rate becomes equal to 1 at age 120 (meaning that there are 1000 deaths in a population of one thousand). In other words, the upper bound of human life is near 120. The existence of this fixed-point has interesting implications. It allows us to predict the form of the relationship between death rates at age 35 and the doubling time of Gompertz's law. In order to test this prediction, we first carry out a transversal analysis for a sample of countries comprising both industrialized and developing nations. As further confirmation, we also develop a longitudinal analysis using historical data over a time period of almost two centuries. Another prediction arising from this fixed-point model, is that, above a given population threshold, the lifespan of the oldest persons is independent of the size of their national community. This prediction is also supported by empirical evidence.

1. Introduction

Among social phenomena there are very few that are governed by laws which are valid with good accuracy in all times and all countries. Gompertz's law is one of them. For a physicist Gompertz's law is fairly unusual because it is an exponential change whose rate itself changes in an exponential way. One will not be surprised that such a process reaches a critical point within a finite time. The present paper draws several implications from this observation.[1]

In 1825 Benjamin Gompertz (1779–1865) derived the law named after him from life tables for the English cities of Carlisle and Northampton. Gompertz's law states that for ages over $t_1 = 35$, the mortality rate $\mu(t) = (1/s)\mathrm{d}s/\mathrm{d}t$, where $s(t)$

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[1] The present paper is the third in a comparative biodemographic investigation which, so far, comprised the following steps: Richmond and Roehner [1,2].

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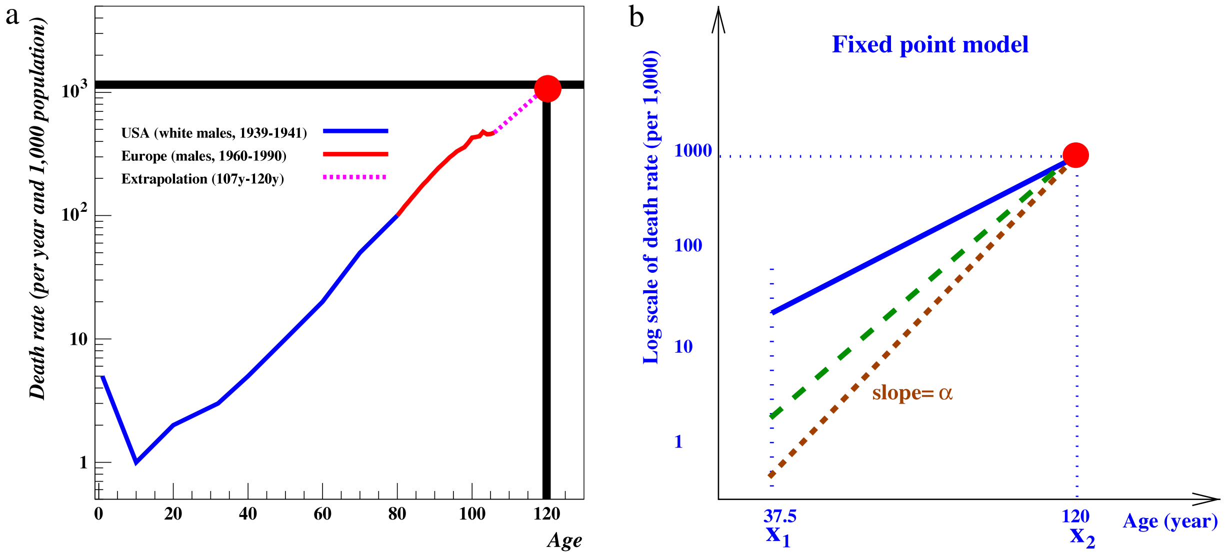

Fig. 1a,b. Gompertz's law. Left: Gompertz's law is characterized by a death rate which increases exponentially with age: $\mu(t) = d_1 \exp(\alpha(t - t_1))$. Here t_1 is about 35 and for $t > t_1$ one gets $\alpha = 0.074$ which corresponds to a doubling time of $\theta = \log 2/\alpha = 9.4$ years. The empirical part of the death rate curve relies on two different sources: (i) first, standard vital statistics; usually such data do not go beyond 95. (ii) the data collected by Arthur Thatcher and his collaborators had for objective to cover ages beyond 95. Yet, beyond the age of 107, the samples become too small even for very large initial populations. That is why, for ages between 107 and 120, we make the conjecture that the line can be extrapolated. This extrapolation leads to the fact that approximately at age 120 the death rate reaches the value 1000 per year and per 1000 persons which means that the population vanishes. This upper bound of 120 agrees fairly well with verified (worldwide) maximum lifespans: Jeanne Calment (122) and Sarah Knauss (119). Right: This graph is a schematic illustration of the fixed-point model. The three Gompertz lines are supposed to correspond to different countries and time periods. Fig. 5a displays a graph of that kind that is based on real data.

Source: Strehler [7], Thatcher et al. [8].

denotes the population of a cohort in the course of time, increases in an exponential manner:

$$\mu(t) = d_1 \exp\left[\alpha(t - t_1)\right] \quad d_1 : \text{ death rate at age } t_1 \geq 35 \text{ years.} \tag{1}$$

From a statistical perspective the exponent α of Gompertz's law is given by the slope of the regression line of the (age, log(death rate)) plot. If law (2) also holds in old age, an obvious implication is that the extinction of a population does not occur asymptotically, but within a finite time interval. This is a consequence of the fact that if for an age t_2 the death rate $\mu(t_2) = (1/s)\,|\Delta s/\Delta t|$ in a unit time interval $\Delta t = 1$ reaches 1, then the number of deaths Δs equals the population $s(t)$ which means that the population vanishes for $t = t_2$. In other words, t_2 represents the strict upper bound of the population's life time. Just to suggest that this occurrence is not purely theoretical, we note that for the population considered in the study by Arthur Roger Thatcher and his collaborators (1999), 79% of the males aged 109 died before reaching 110.

The paper proceeds as follows.

- In the following section, we use the fixed point model to derive the relationship between the death rate at age 37.5 and Gompertz's exponent α.
- In the three subsequent sections we test the accuracy of this relationship by two different methods and we propose an interpretation.
- In the penultimate section we test another prediction of our fixed point model.
- Finally, in the concluding section we discuss a number of related issues including one raised in a recent paper about the connection between infant mortality and aging.

Before we start our analysis it may be of interest to point out that among econophysicists there has been in recent years a growing interest for demographic problems [3–6]. One important reason is the low level of noise in demographic data. Whereas stock markets are characterized by a great heterogeneity in agents (from day-traders to long-term stock holders) as well as in types of stocks (from stocks that are heavily traded to stocks which are rarely traded), human beings are much more homogeneous with respect to their response to biological factors.

2. Derivation of the prediction for Gompertz's exponent

The data for Gompertz's law up to age 106 are summarized in Fig. 1a. The data come from two sources. For ages up to 80 (or sometimes 95) the death rates can be taken from standard death rate statistics as published in all countries. For ages up to 106, the data are taken from Thatcher et al. (1999) [8] based on a global sample of 13 countries over several years. The monograph actually gives death rates up to age 113, but beyond age 106 the numbers involved become very small which gives rise to substantial random fluctuations.

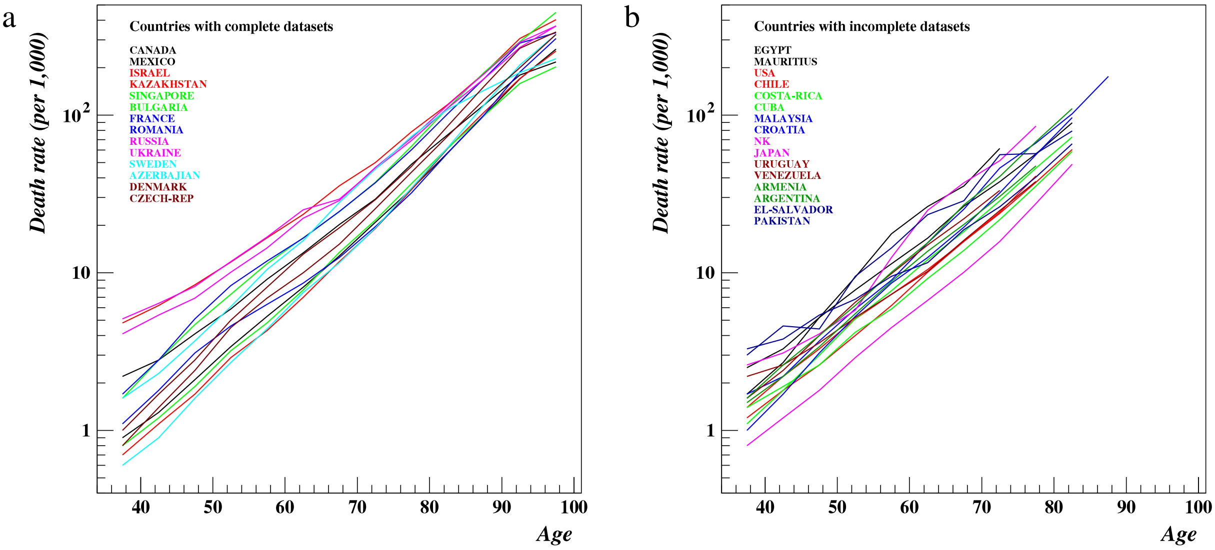

Fig. 2a,b. Gompertz's law in two sets of countries. Left: Group (*A*) of countries which give death rates for ages up to 95–99 year old. Right: Group (*B*) of countries which provide death rate data only for shorter age intervals. In Fig. 3a, b and Fig. 4a, b complete and incomplete data will also be on the left and right respectively. Ideally all lines are expected to converge toward the end-point (120,1000). Not surprisingly, this convergence is clearer in group *A* than in group *B*.

Source: United Nations Demographic Yearbook, 2011.

Such fluctuations explain why studies using national death rate data for persons over 100 lead to fairly shaky results. Some studies (e.g. Ref. [7] or [4]) display a deceleration effect,[2] whereas in others (e.g. the graph for the United States in 2003 that accompanies the Wikipedia article entitled "Gompertz–Makeham law of mortality") there appears to be a death rate acceleration. For the age interval 80–106 [8] show clearly that Gompertz's law continues to hold. Beyond 106 there is still no convincing evidence. One will have to wait until studies similar to the Thatcher study are performed in countries with large populations such as China or India. In the present paper we conjecture that Gompertz's law can be extrapolated beyond 106 years. This puts the term of human life at 120 years.[3]

Fig. 1b illustrates the simple idea upon which the present study is based. We refer to it as the "fixed-point model".

If over age $t = 35$ Gompertz's law holds for all countries and if 120 years is the end point of human life, then the equation of any straight line which summarizes Gompertz's law for a specific country will be determined by the initial death rate from which it starts. If one denotes the logarithm of the death rate by y, according to a standard formula, this equation reads:

$$y = \alpha(t - t_1) + q, \quad \alpha = \frac{y_2 - y_1}{t_2 - t_1}, \ q = \frac{t_2 y_1 - t_1 y_2}{t_2 - t_1} \ y = \log(\text{death rate}), \ t = \text{ age}.$$

For the initial age t_1 we selected an age such that in any country Gompertz's law holds for $t \geq t_1$. Observation shows (see Fig. 2a, b) that this is the case for the age intervals above (and including) 35–39. This leads to a value of t_1 equal to the mid-point of this interval, i.e. $t_1 = 37.5$ years. Then, replacing t_1, t_2 by 37.5 and 120, and y_2 by log 1000, one gets the following relationship between the exponent α and $\log d_1$.

$$\alpha = \frac{1}{t_2 - t_1} \log d_1 + \frac{\log 1000}{t_2 - t_1} = -0.0121 \log d_1 + 0.084. \tag{2}$$

In the numerical form of this relationship it is supposed that α is expressed in year^{-1} and d_1 in number of deaths per year and per 1000 population.

3. Test of the prediction for Gompertz's exponent

3.1. Transversal analysis

In order to study the variability of Gompertz's law across countries, we need to set up an appropriate "experiment". The first step is to identify a source providing international data and the Demographic Yearbooks published by the United Nations meet this our need.

2 Such a deceleration is predicted by the Penna model of mutation-accumulation, or at least by one form of this multi-faceted model.

3 If instead there is a deceleration after 106 the term of life would be slightly beyond 120 but this would not change the main argument used in this paper. Incidentally, it can be observed that, according to the Torah, Moses died at the age of 120, a fact which is reflected in the Yiddish anniversary wish "Bis hundret und tzvanzig". In contrast, earlier Patriarchs are reported to have had much longer lives; for instance, it is stated that Methuselah lived until the age of 969.

Table 1
Relationship between initial death rates d_1 and Gompertz's exponents: $\alpha = -a \log d_1 + b$.

	$100 \times a$	$100 \times b$
Prediction of the fixed-point model	1.21	8.37
Group A (complete data)	1.33 ± 0.20	9.54 ± 0.13
Difference with respect to prediction	10%	14%
Group B (incomplete data)	1.34 ± 1.0	9.37 ± 0.41
Difference with respect to prediction	11%	12%

Notes: For clarity the table gives a, b multiplied by one hundred. Comparison of the error bars (probability level of 95%) shows that for a as well as b the accuracy of the observations in group A is five times better than in group B.

Secondly, for an accurate definition of the relationship between d_1 and α we need observations for a wide range of d_1 values. So we cannot restrict ourselves to data from industrialized countries because in all these countries d_1 will be limited to a narrow range of small values, basically of the order of 1 per 1000.

Furthermore we wish to study the convergence of death rates toward the fixed-point at age 120 so we need death rates for age groups which are as close as possible to 120. Finally, since at this point we are not particularly interested in the difference between male and females we shall use both-sex data.

By accessing the UN Yearbook of 2011 one observes immediately two things:

- While *numbers* of deaths by age are given for many countries, death *rates* are given for only about one third of the countries.
- In the subset of countries for which rates are available only about one third of them provide data up to the age group 95–99. For the others, the data are given only up to 70, 80 or 85 depending on the country. This will lead us to distinguish two groups of countries: group A which contains all countries providing data up to age-group 95–99 (such data will be referred to as "complete data") and group B which will contain all other countries (it will be referred to as the "incomplete data" group).
- In group A there are countries with death rates at age 37.5 that are as low as 0.6 per 1000. At the other end of the spectrum high death rates can be expected in developing countries particularly in African countries. However, among the African countries with sizable populations there are only 4 for which death rates are given, namely: Egypt, Sierra Leone, Swaziland and Zimbabwe. Egypt was included in group B but its death rate, d_1, at age 37.5 is 1.7 which is not particularly high. In the other three countries $d_1 = 11.3,\ 36.3,\ 38.9$. Such values would be quite useful but unfortunately the death rate series do not appear reliable. This can be seen in two ways. First, by the fact that they are labeled by the UN as I (instead of C which means "complete") and secondly because the death rates do not increase in a monotonic way. Thus, for Swaziland the death rates in the age groups 70–74 and 75–79 are 48.0 and 45.4 respectively which, although not altogether impossible, is highly unlikely.

In conclusion, we expect fairly accurate results in group A but poorer accuracy in the results for group B.

3.1.1. Results

The results are summarized in Fig. 2a, b. First one observes that for each and every country, Gompertz's law holds with high accuracy. More precisely, for the data of group A the correlation between age and logarithm of death rate is always higher than 0.995. Yet, the slopes may differ substantially. This leads to the two following questions.

(1) Is there a regularity in the variations of the slopes or are they just random?
(2) If there is a regularity does it follow the fixed-point model?
(3) If the data are indeed well described by the fixed-point model, how can one explain this effect in biological terms?

Fig. 3a, b and Table 1 answer questions 1 and 2. They show that there is indeed a relationship between *alpha* and $\log d_1$ of which the fixed point model provides a fairly accurate prediction.

Before turning to question 3 we describe a second test of questions 1 and 2 that is based on longitudinal analysis.

3.2. Longitudinal analysis

Western countries began to collect reliable demographic statistics in the mid-19th century. As death rates at age 35–39 were substantially higher at that time can we possibly use such data to explore the region of high d_1 that was out of reach in our cross-national analysis? As a test-case we consider France. In Ref. [9] one can find death rate data by age-group from 1806 on.

$$1806 : d_1 = 14.8, \qquad 1836 : d_1 = 11.8, \qquad 1866 : d_1 = 11.3, \qquad 2005 : d_1 = 1.1.$$

As the data for 1806 may be somewhat less reliable than later ones, we selected 1836, 1866 and 2005. Our purpose in taking 1836 and 1866 in spite of the fact that they have almost the same value of d_1 is to control the reliability of the data.

This longitudinal analysis leads to the results summarized in Fig. 5a, b.

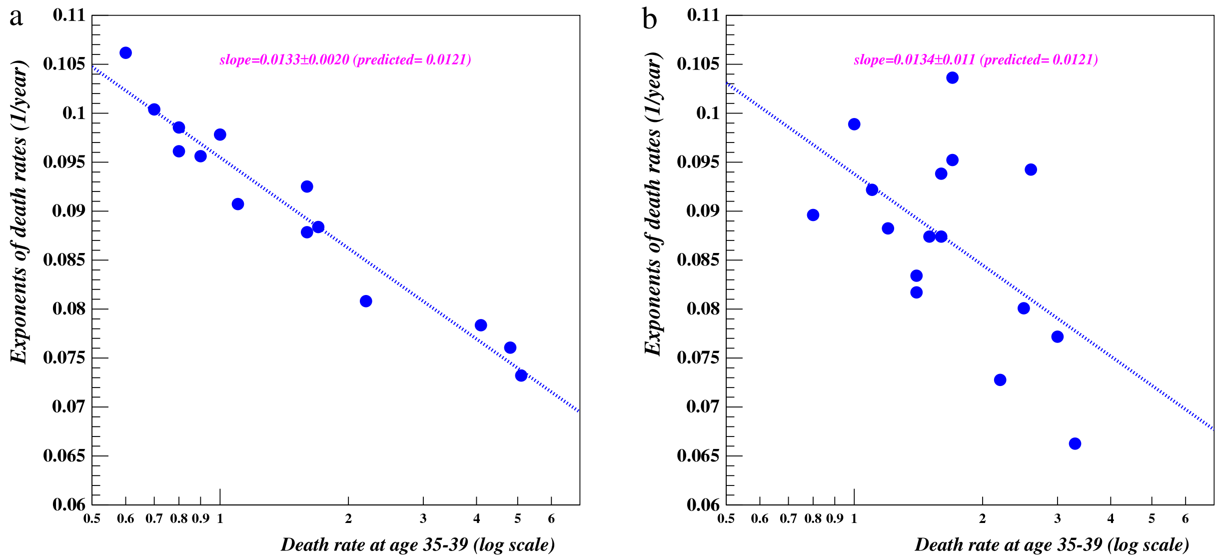

Fig. 3a,b. Exponents of Gompertz's law as a function of death rates in the 35–39 age-group ($\log d_1$). Left: Group *A* of countries with complete datasets. The correlation ($\log d_1, \alpha$) is -0.970. Right: Group *B* of countries with incomplete datasets. The correlation is -0.55. Although the slopes of the regression lines are almost the same in the two groups, the accuracy of their measurement is about 5 times better in group *A* than in group *B*. The exponents predicted by the fixed-point model are within the error bars of the observations.

Source: United Nations Demographic Yearbook, 2011.

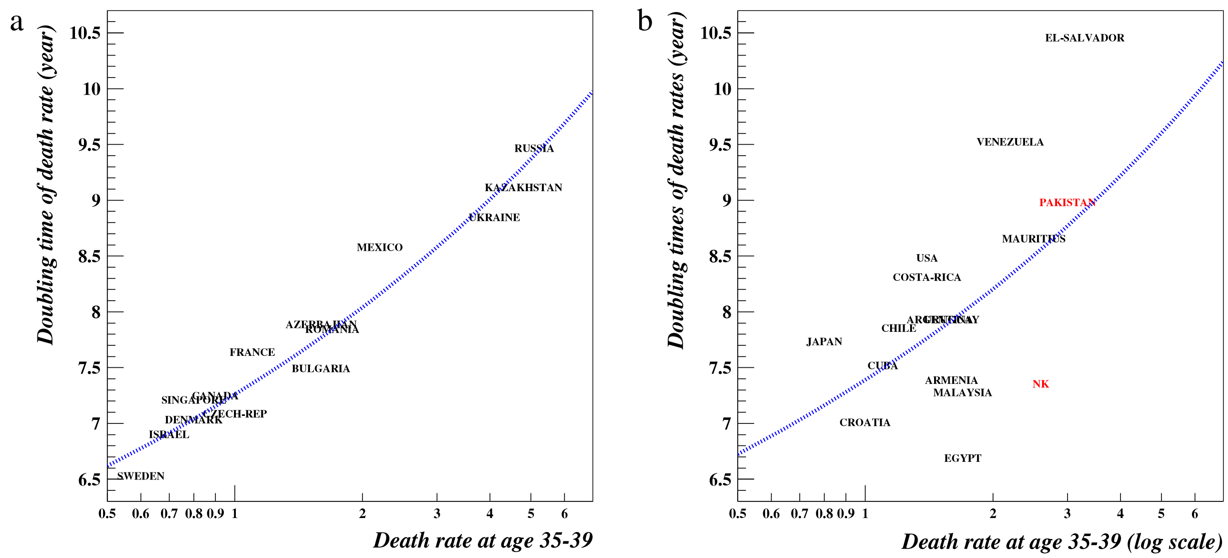

Fig. 4a,b. Doubling times θ of death rates as a function of death rates in the initial 35–39 age-group (d_1). Left: Group *A* of countries with complete datasets. Right: Group *B* of countries with incomplete datasets. According to the fixed-point model there should be an hyperbolic relationship between the two variables: $\theta = \log 2/(-a \log d_1 + b)$. The non linearity of the relationship would become more obvious for higher initial death rates. In some African countries (e.g. Sierra Leone or Zimbabwe) initial death rates as high as 30 per 1000 were reported but the demographic data of such countries appear fairly unreliable (see text). The names of North Korea and Pakistan are written in red because the data provided by these countries are acknowledged to be incomplete (even for the restricted age range for which data are available). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Source: United Nations Demographic Yearbook, 2011.

4. Interpretation: filter effect and "natural" death rate levels

The simplest interpretation which comes to mind for the results described in the previous section relies on a filter effect. There are two steps in this explanation.

First, we note that the death rate at age 37.5 is not an isolated number but is related to the rates in other age intervals. Thus, for a sample of countries the infant mortality rate $d(0–1)$ and the adult death rate $d(35–39)$ have a cross-correlation

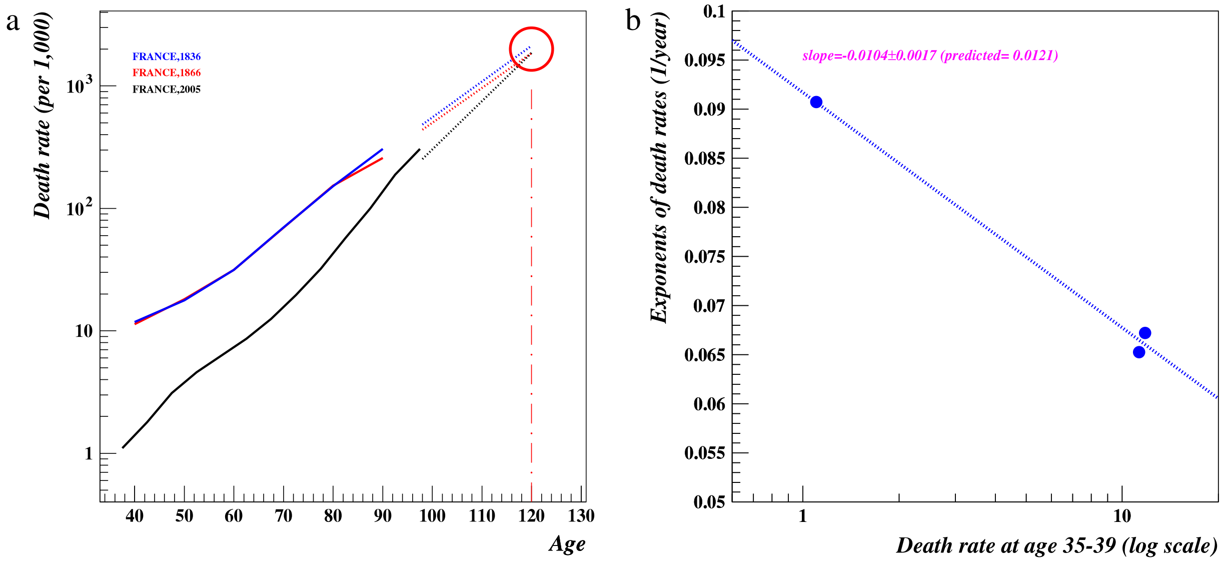

Fig. 5a,b. Gompertz's law in France in different years. Left: The dotted straight lines beyond 95 are the regression lines. Ideally one expects all lines to converge toward the end-point (120,1000). In fact, the center of the red circle around the convergence point is somewhat higher than 1000. The fact that the lines for 1836 and 1866 are almost identical controls the reliability of the data. Right: Regression line for (d_1, α) and comparison with the predicted slope. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Source: 2005: United Nations demographic yearbook, 2011; 1836, 1866: INSEE [9].

of the order of 0.7; $d(35–39)$ has also a correlation of same magnitude with $d(1–4)$. This means that a high $d(35–39)$ was preceded by a range of fairly high death rates at a younger age. It should be observed that at first sight such correlations are rather unexpected since for such age intervals the causes of death are fairly different. For the young, say for age below 5, the main causes of death are diseases whereas for the age group 35–39 the main causes of death are external factors, e.g. accident, suicide, homicide. The fact that there is nevertheless a correlation is probably related to the nature of the social organization. Low infant mortality implies a rich country which in turn implies well organized transports (few traffic accidents) and little violence.

Now, the meaning of the filter effect becomes clear. In a society characterized by a high $d(35–39)$ there is also a high infant mortality which means that persons who for various reasons are "fragile" (e.g. low immunity, heart malformation) will die in the early years of their lives. It follows that the survivors will be less susceptible to the diseases which usually come with age. This implies a death rate that will increase relatively slowly with age.

A second mechanism may also be at work which is complementary to the previous one but nevertheless distinct. It relies on the idea of a "natural death rate" brought about by aging. More precisely, it seems that medical progress cannot drastically change the death rates for ages over 60. Whereas during the past century infant mortality has been divided by over 30, the death rates due to aging have not been reduced to the same extent. Thus, in advanced countries where $d(35–39)$ is small, aging will produce an increase toward "natural" death rate levels that will be faster than in countries where $d(35–39)$ is already large.

Through a close examination of the historical records of advanced countries it should be possible to estimate the respective weights of these two mechanisms but this will be left for a subsequent paper.

5. 2nd prediction: impact of sample size on maximum life span

5.1. The Wall Effect

The abrupt collapse of any population sample, that will be called here the "Wall Effect", was already illustrated in Fig. 1 by the fact that the survivorship function becomes strictly equal to zero for age $t = 120$. Here, however, the implication will be stated in the language of probability theory. The survivorship curve describes the population of a cohort in the course of time. If the initial population is normalized to 1, this curve becomes equivalent to the probability for an individual to reach a given age t that is to say $P\{X \geq t\} = 1 - F(t) = G(t)$, where $F(t)$ is the cumulative probability distribution function and $G(t)$ the complementary distribution function. Thus, $f(t) = -\mathrm{d}G/\mathrm{d}t$ where $f(t)\mathrm{d}t = P\{t \leq X \leq t + \mathrm{d}t\}$ is the probability density function. It gives the number of people in a cohort whose lifespan is in the interval $(t, t + \mathrm{d}t)$. The function $G(t)$ for the lifespan in France is given in Fig. 6a.

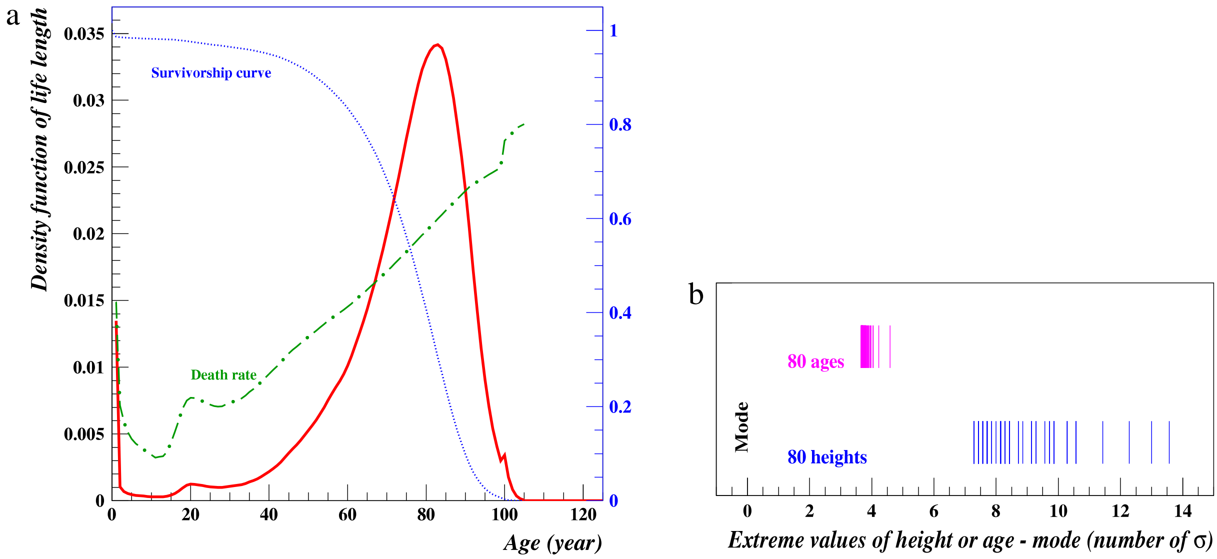

Fig. 6a,b. Comparison of extreme values of height and age. Left: Density function of the duration of human life (both sexes together). The survival curve $s(t)$ is identical to the complementary distribution function: $s(t) = 1 - F(t)$. Thus, the density function is the opposite of the derivative of the survivorship curve: $f(t) = \mathrm{d}F/\mathrm{d}t = -\mathrm{d}s/\mathrm{d}t$. The death rate curve is the opposite of the derivative of the logarithm of the survivorship curve: $\mu(t) = (1/s)\mathrm{d}s/\mathrm{d}t == -d\log[s(t)]/\mathrm{d}t$. Right: Comparison between 80 top values for height (men) and length of life (women). The vertical lines show the differences (expressed in number of σ) between individual top values and the mode (i.e. peak value) of the distribution. In each case the standard deviation σ was computed from the values in the vicinity of the peak. The wall effect for maximum ages is quite apparent. Note that for height the small number of vertical lines is due to the fact that many realizations correspond to identical positions because heights are expressed in centimeters rather than in millimeters. Incidentally, it can be noted that the extreme values of height do *not* follow a Gaussian distribution for in a real Gaussian (with here $\sigma \simeq 7$ cm) existence of heights of 230 cm (i.e. 7.6σ) would require a population of $1/G(7.6) = 10^{14}$ people. The same observation can be made regarding the existence of dwarfs.

Source: Survivorship curve: Official mortality table (TD73-77) used by insurance companies in France. Extreme values: Wikipedia lists of tallest and oldest persons.

The fact that: $f(t) \equiv 0$ for any $t \geq 120$ has two consequences.

- Maximum values of lifespan will be squeezed within a short interval ending at 120. For the purpose of comparison Fig. 6b shows also maximum values for height. We chose this variable because it is usually considered as the archetype of a Gaussian distribution. Incidentally, while indeed true for heights within 2 or 3 σ around the mean this is no longer true for the tails of the distribution.
- Extreme lifespan values are independent of sample size. For any random variable whose distribution function $G(t)$ tends toward 0 (yet without reaching 0) as t increases, the largest realizations are conditioned by the size of the sample. Thus, if for the height (expressed in cm) one has $G(200) = 10^{-3}$ one will need a sample of at least 1000 individuals in order to have a chance to get one person with a height over 200 cm. With this logic (and leaving aside genetic factors) the height of the tallest persons will be larger in the United States than in Iceland whose population is 1000 times smaller. On the contrary, for age, above an appropriate threshold p_0, the size of the sample will not play any role. This prediction of the fixed-point model is easy to check. Tables of oldest persons by country can be found on Wikipedia. Here are some cases. The first number gives the population P in millions, the second the age t of the oldest person.

Belgium: 11, 112.5; Denmark: 5.6, 115.7; France: 66, 122.4 Germany: 80, 115.2; Iceland: 0.33, 109.8; Ireland: 4.6, 113.4; Italy: 60, 115.7; Moldova: 3.5, 114.8; Russia: 143, 113.0; United States: 319, 119.3.

There is a correlation of about 0.70 between $\log P$ and t but the correlation vanishes for $t > 10$ million which means that over $p_0 \sim 10$ million there is no longer any connection between the size of the sample and the longest lifespan.

6. Conclusion

6.1. Extension of the fixed point model to other species

The fixed-point model developed in this article is based on two observations.

(1) The validity of Gompertz's law for ages over 40.

(2) The fact that the upper bound of human life is about 120 years.

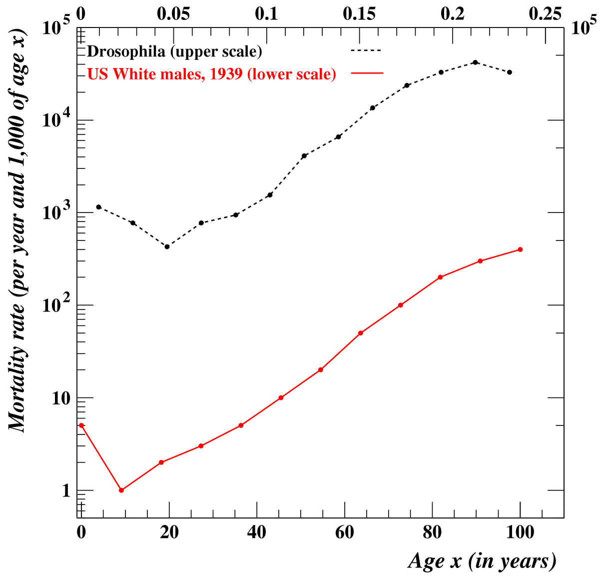

Fig. 7. Age-specific mortality rates for humans and drosophila. Gompertz's law holds in both cases. As seen at the beginning of the paper, the leveling off observed in humans should be attributed to statistical fluctuations due to the small numbers of the survivors. For the leveling off of the drosophila curve we do not yet know whether it is spurious or real.

Source: Strehler [7], Miyo and Charlesworth [10], Wang et al. [11].

So far, the second point remains a conjecture. It was tested through its consequences but direct verification will become possible only once reliable data become available for populous Asian countries. Data today for present-day centenarians pose a challenge since their birth took place in a time when vital statistical records may not have been accurate.

Naturally, the analysis developed here for humans can be extended to any other species for which (i) Gompertz's law holds and (ii) an upper bound of the life span can be demonstrated. As an illustration let us consider the drosophila fruit flies. As shown in Fig. 7, their age-specific mortality rates follow Gompertz's law.

One may wonder whether the leveling off seen in the drosophila curve is real or rather due to statistical fluctuations. There is no doubt that the question could be settled by performing repeated observations on large populations. Anyway, it should be observed that there can be an upper bound of the life span even if there is a leveling off. Only a steady decrease of $\mu(t)$ would prevent it from reaching the terminal rate of 1000 per 1000.

What can be expected from the observation of populations of drosophila? The first question is of course to see whether there is a fixed point or not and if there is one how robust it is with respect to changes in the parameters such as temperature, light, male/female ratio which shape the living conditions of the drosophila. It is obvious that if living conditions are not appropriate the fixed-end point will remain out of reach. On the contrary, conditions under which some members of a cohort will reach the fixed-end point will define the "envelope" of acceptable living parameters.[4] Within this envelope it will be possible to study the influence of various parameters, including "social" variables such as the male/female ratio (in this respect see Refs. [12,13]). In this way, it may be possible to test the relationship between d_1 and α in an experimental way. That should lead to clearer and more accurate results than the observational method that one must use for human populations.

Naturally, the drosophila fruit flies are not the only living organisms for which Gompertz's law holds. For instance, it was shown by Raymond Pearl [14] that the lifetables of flour beetles are fairly similar to human life tables.

6.2. Comments on earlier papers

In contrast with common practice, we prefer to present a discussion of other papers in our conclusion rather than in the introduction. The reason is obvious. Once the problem that we wish to solve has been clearly explained, it becomes easier to describe the contributions of other papers.

The fact that the slope of Gompertz's law is country and period dependent has been observed as soon as international demographic data became available (e.g. Refs. [15,16]). However, the explanations put forward in such studies have been different from the one given in the present paper because the validity of Gompertz's law in the age range 95–106 which is a key-element in our explanation was clearly established only in 1999 through the work of Thatcher and his collaborators.

[4] The word "envelope" is used in the same sense as for the envelope of flight parameters of an aircraft.

In the following lines we briefly discuss two studies: Strehler [17,7] and Beltrán-Sánchez et al. [18]. A third one, Finch [19], is mentioned in the reference section.

The international data used in Refs. [17,7] are from the UN Demographic Yearbook of 1955 and concern 32 countries. Quite surprisingly the authors study the regression between α and the logarithms of extrapolated mortality rates at age 0,[5] Considering that the intercept was not drawn from an independent source but from the regression line itself is this not a case of circular reasoning? In the present paper the death rates at age 37.5 were taken from death rate data *independently* of the regression procedure.

The objective of the paper by Beltrán-Sánchez et al. is related to but different from the purpose of the present paper. These authors wish to determine if in historical perspective there is a connection between infant or child mortality, mid-age mortality and the slope of Gompertz's law. This is indeed an important question. The authors show that between 1800 and 1915 there is indeed a connection between early life mortality and late life mortality. Why did they limit their study to 1800–1915? One can guess that they left aside following decades because during the 20th century medical progress brought about a dramatic decline in early life mortality which would have made the effect they wanted to study all too obvious. By limiting themselves to the period prior to 1915, they have a better chance to observe the biological aspect independently of the impact of medical advances. Another way to achieve that objective would have been to study this effect in animal populations. This brings us back to the proposition that if carried out in parallel with demographic studies, the investigation of animal populations can provide valuable additional insight.

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[5] The authors describe their procedure as follows: Each country's mortality rates (for various ages) were "plotted on semi-log paper. A straight line was drawn between points from 35 to 85 (or 50–70 if large departures from linearity occurred), and the slopes and intercepts were measured. Only a few countries, whose Gompertz plots exhibited great irregularities were excluded". Thus, it seems clear that the intercepts were not drawn from an independent source.