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Landauer's Bound and Maxwell's Demon

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Abstract. We summarize recent experimental and theoretical progress achieved in the physics of information. We highlight the intimate connection existing between information and energy from Maxwell's demon and Szilard's engine to Landauer's erasure principle. We will focus mainly on experiments on classical systems and we will shortly discuss a few aspects of quantum systems. We conclude by discussing applications in engineering and biology.

Contents

1 Introduction

This review summarizes the contents of several articles [1, 2, 3, 4, 5], that we wrote on the experimental aspects of the connections between thermodynamics and information. We first define the theoretical framework of this connection by presenting a short historical review starting from the article [6] of Rolf Landauer in which he argued that information is physical. Since information is processed in physical devices, he concluded that information has to obey the laws of physics, and in particular the laws of thermodynamics. Information is thus stored in physical systems, such as books or memory sticks, and transmitted by physical means, for instance with the help of electrical or optical signals. But what is 'information'? A simple, intuitive answer is 'what you don't already know'. If someone tells you that the earth is spherical, you surely would not learn much: this message has low information content. However, if you are told that the oil price will double the day after tomorrow, assuming for a moment this to be true, you would learn a great deal : this message has hence high information content. Mathematically, the amount of information is quantified by the so-called information entropy H introduced by Claude Shannon in 1948; the larger the entropy, the bigger the information content [7]. The simplest device to store information is a system with two distinct states, for example up/down, left/right or magnetization/no magnetization. If the system is known to be with probability one in one of either states, probing the system will not reveal any new information, and the Shannon entropy is zero. On the other hand, if the two states can be occupied with probability one-half, and the actual state is therefore initially undetermined, an examination of the system will provide information about the state it is in. In this case, the Shannon entropy is equal to $\ln(2)$. This value corresponds to the smallest amount of information and is called a bit. A two-state system can thus store up to one bit of information.

The second law of thermodynamics, as formulated by Rudolf Clausius in 1850, is based on the empirical observation that some processes only occur spontaneously in one preferred direction [8]. Everyone who forgot a cup of hot tea on a table has noted that heat flows by itself from a hotter (the cup) to a colder body (the room), and never the other way around. Heat flow is therefore said to be irreversible. Clausius characterized the irreversibility of natural macroscopic processes by defining the thermodynamic entropy S, a quantity that is not conserved, in contrast to energy, but can only increase in isolated systems. This asymmetry in the change of entropy imposes restrictions on the type of physical phenomena that are possible. Similarly, the application of the second law of thermodynamics to information sets limitations on information processing tasks such as transmission or erasure. More general questions address the thermodynamic consequences of information gain. In particular, whether it is possible to extract useful mechanical work from a system by observing its state, and if yes how much. And at the more fundamental level: are thermodynamic and information entropies related [2, 9]?

1.1 Maxwell's demon and Szilard's engine

The first hint of a connection between information and thermodynamics may be traced back to James Clerk Maxwell's now famous demon introduced in 1867 [10, 11, 12]. The demon is an intelligent creature able to monitor individual molecules

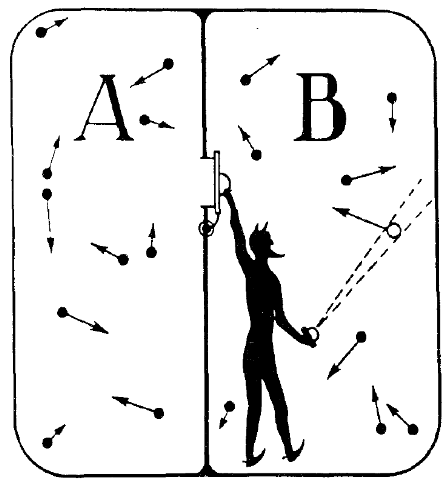

Figure 1: Maxwell's demon. By detecting the positions and velocities of gas molecules in two neighboring chambers and using that information to time the opening and closing of a trapdoor that separates them, a tiny, intelligent being could, in theory, sort molecules by velocity. By doing so, it could create a temperature difference across the chambers that could be used to perform mechanical work. If the trapdoor is frictionless, the sorting requires no work from the demon himself, in apparent violation of the second law of thermodynamics (drawn by Claire Lebeau)

of a gas contained in two neighboring chambers initially at the same temperature, as shown in Fig. 1. The temperature of the gas is defined by the mean kinetic energy of the molecules and is hence proportional to their mean-square velocity. However, not all the particles will have the same velocity. Some of the molecules will be going faster than average and some will be going slower. By opening and closing a molecular-sized trap door in the partitioning wall, the demon collects the faster molecules in one of the chambers and the slower ones in the other. The two chambers now contain gases with different mean-square velocities and hence different temperatures. This temperature difference may be used to run a heat engine and produce mechanical work. By gathering information about the position and velocity of each particle and using this knowledge to sort them, the demon is therefore able to decrease the entropy of the system and convert the acquired information into energy. The problem is that the demon, assuming a frictionless trap door, is able to do all this without performing any work himself, in apparent violation of the second law of thermodynamics. The proper resolution of this paradox took 115 years.

A simplified one-particle engine has been suggested by Leo Szilard in 1929 [13]. In this setup, schematically shown in Fig. 2, the gas consists of a single molecule and the wall separating the identical chambers is replaced by a moving piston to which a weight can be attached. We now have a two-state system very similar to the one discussed above. Initially, the particle has a probability of one half to be in one of the two chambers. By looking into the container the demon acquires information about the actual state of the system, learning what he did not know before. If the molecule is found in the right chamber, the weight is attached to the right-hand side of the piston which is then released from its former position. During the expansion of the gas, the piston is pushed to the left and the weight is pulled upwards, performing work against gravity. The piston is attached to the left-hand side of the piston when the molecule is observed in the left chamber. The second law of thermodynamics limits the maximum amount of work that can be produced by the Szilard engine to $k_B T \ln(2)$, where k_B is the Boltzmann constant and T the temperature of the gas. This corresponds to the maximum amount of energy that can be obtained by converting one bit of information, and is historically the first clear statement of the relationship between information and energy. In modern language, this result further implies that information and thermodynamic entropies are equal, $S = k_B H$, up to the multiplicative factor k_B introduced for dimensional reasons (the Shannon entropy H is dimensionless).

1.2 Landauer's principle and Bennett's resolution

It is useful to distinguish two complementary aspects: the first one is information gain, as we have just discussed with Maxwell's demon, the second one is information erasure, which has been investigated from a thermodynamic point of view by Landauer in 1961. Let us again consider a two-state system and let us assume that it initially stores one bit of information, that is, the two states are occupied with equal probability one-half. This bit may be erased by resetting the system to one of the states, which will then be occupied with unit probability, a situation that corresponds to a zero Shannon entropy. By applying the second law of thermodynamics, Landauer demonstrated that information erasure is necessarily a dissipative process: the erasure of one bit of information is accompanied by the production of at least $k_B T \ln(2)$ of heat into the environment. This result is known as Landauer's erasure principle. It emphasizes the fundamental difference between the process of writing and erasing information. Writing is akin to copying information from one device to another: state left is mapped to left and state right is mapped to right, for example. This one-to-one mapping can be realized in principle without dissipating any heat (in statistical mechanics one would say that it conserves the volume in phase space). By contrast, erasing information is a two-to-one transformation: states left and right are mapped onto one single state, say right (this process does not conserve the volume in phase space and is thus dissipative).

Landauer's principle played a central role in solving the paradox of Maxwell's demon. In 1982 Charles Bennett noted that the demon has to store the information he acquires about the gas molecules in a memory [14]. After a full information gathering energy producing cycle, this memory has to be reset to its initial state to allow for a new iteration, and its information content has thus to be erased (a similar argument was put forward by Oliver Penrose in 1970 [15]). According to Landauer's principle, the erasure process will dissipate an amount of energy that is always larger than the quantity of energy produced by the demon during one cycle. The demon has consequently to pay an energetic price to sort the molecules and have heat flow from the colder chamber to the hotter chamber, in full agreement with the second law of thermodynamics. Before Bennett's resolution, it was often believed, following arguments put forward by Leon Brillouin and Dennis Gabor, that it was the energetic price of the measurement, that is, of the act of gathering information, that would save the second law [16]. However, as shown by Bennett, there is no

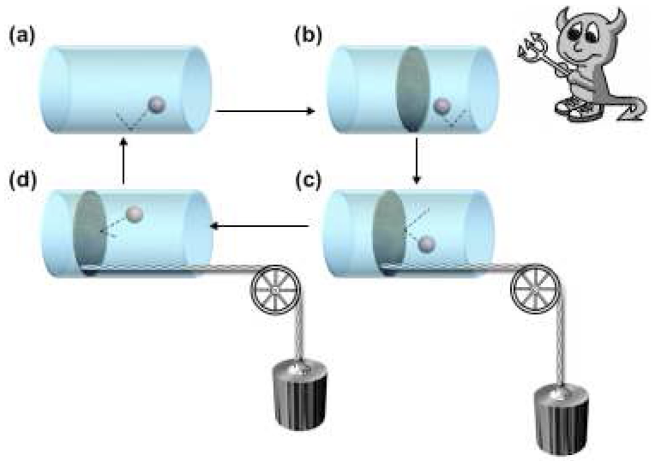

Figure 2: Szilard's engine. A crafty observer can turn a single particle in a box into an engine that converts information into mechanical work. If, say, the particle is found on the box's left-hand side, the observer inserts a movable wall and attaches a weight to its left side. The free expansion of the one-particle gas pushes the wall to the right, lifts the weight, and thereby performs work against gravity (adapted from Ref. [12]).

fundamental energetic limitation on the measurement process, which like the copy operation, may in principle be performed without dissipation, in stark contrast to erasure.

Box 1: Landauer's erasure principle

Landauer's principle can be seen as a direct consequence of the second law of thermodynamics. Consider a system (SYS) coupled to a reservoir (RES) at temperature T. According to the second law, the total entropy change for system and reservoir is positive: $S_{\rm TOT} = S_{\rm SYS} + S_{\rm RES} \geq 0$. Since the reservoir is always at equilibrium, owing to its very large size, we have following Clausius, $\Delta S_{\rm RES} = Q_{\rm RES}/T$. In other words, the heat absorbed by the reservoir satisfies $Q_{\rm RES} \geq T\Delta S_{\rm SYS}$. For a two-state system that stores one bit of information, there are initially two possible states that can be occupied with probability one half, and the initial Shannon entropy is $H_i = \ln(2)$. After erasure, the system is with unit probability in one of the states and the final Shannon entropy vanishes $H_f = 0$. The change of information entropy is thus $\Delta H = -\ln(2)$. During this erasure process the ability of the system to store information has been modified. By further using the (assumed) equivalence between thermodynamic entropy S and information entropy H we can write $\Delta S_{\rm SYS} = k_B H = k_B \ln(2)$. We hence obtain $Q_{\rm RES} \geq k_B T \ln(2)$, showing that the heat dissipated into the reservoir during the erasure of one bit of information is always larger than $k_B T \ln(2)$.

2 Experimental implementations

For almost a century and a half, the demon belonged to the realm of a gedanken experiment as the tracking and manipulation of individual microscopic particles was impossible. However, owing to the remarkable progress achieved in the last decades, such experiments have now become feasible. Just to give a hint on what can be done, we will discuss in the following sections several experimental realizations of Maxwell's demon and Szilard's engine, as well as several verification of Landauer's principle.

2.1 Experiments on Maxwell's demon

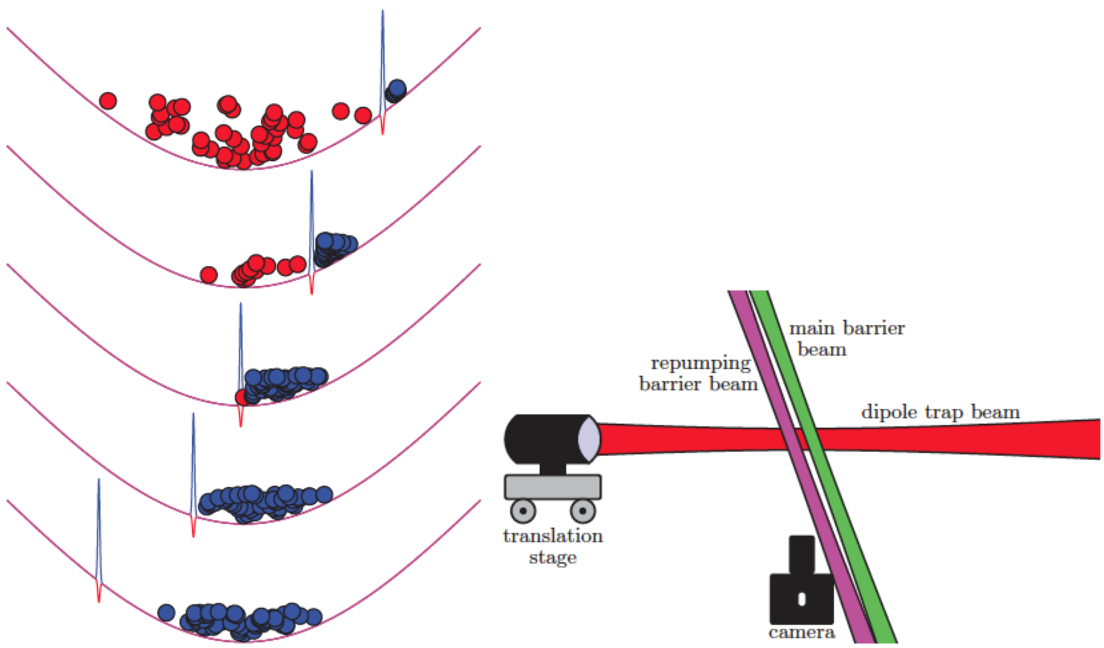

Figure 3: Using a Maxwell's demon to cool atoms. A pair of laser beams can be tuned to atomic transitions and configured to create a one-way potential barrier; atoms may cross unimpeded in one direction, from left to right left in this figure, but not in the other. Left panel : when the barrier is introduced at the periphery of the trapping potential, (right side) the atoms that cross the barrier will be those that have converted nearly all their kinetic energy to potential energy, in other words, the cold ones. By slowly sweeping the barrier (from the right to the left) across the trapping potential, one can sort cold atoms (blue) from hot ones (red), reminiscent of Maxwell's famous thought experiment, or cool an entire atomic ensemble. Because the cold atoms do work against the optical barrier as it moves, their kinetic energy remains small even as they return to the deep portion of the potential well. Right panel: schematic representation of the optical set-up showing the optical trap (red beam), the translational stage and the two beams one way barrier (adapted from Ref. [17]).

The first realization of a Maxwell demon was used to cool atoms in a magnetic trap. An ensemble of atoms is first trapped in a magnetic trap (see Fig. 3) [18]. A one way barrier (which plays the role of the demon) sweeps the magnetic trap from the right to the left, starting at a very large value of the potential. The atoms reaching this position have transformed almost all their kinetic energy in potential energy and are, therefore, very cool. These atoms go through the barrier but they cannot come back, i.e. the barrier behaves as an atom-diode [18, 19, 17]. Thus the hot atoms are on the right and the cold atoms are on the left. At the end of the process when the sweeping one-way barrier reaches the bottom of the magnetic potential all of the atoms are cooled down. The one way barrier is composed by two laser beams suitable tuned to atomic transitions. With reference to fig. one of the two lasers is on the left of the barrier and forces the atoms in an excited state. The frequency of the second laser, which is on the right of the barrier, is tuned in such a way that it has no effect on the atoms in the excited state and it repels the atoms in the ground state. Thus the atoms coming from the right, which are prepared in the excited state, go through the barrier and relax to the ground state by emitting a photon. Instead the atoms coming from the left, which are in the ground state, encounter first the barrier and remain trapped because they are repelled. Where does the connection with Maxwell demon come from? Indeed each time that an atom looses a photon the entropy of the light shining the atoms increases because before all the photons were coherently in the laser beam (low entropy state) and now the emitted photons are scattered in all directions (high entropy state). This entropy is related to an information entropy because each time that a photon is emitted we know that an atom has been cooled. It can be shown that indeed this gain of entropy is larger than the reduction of entropy produced by the cooling of the atomic cloud. It is important to notice that in this example the demon has not to be an intelligent being but it is just a suitable tuned device which automatically implements the operation.

2.1.1 *The Szilard engine: work production from information*

A Szilard engine has been realized in 2010 by using a single microscopic Brownian particle in a fluid and confined to a spiral-staircase-like potential shown in Fig. 4 [20]. Driven by thermal fluctuations, the particle performs an erratic up and down motion along the staircase. However, because of the potential gradient downwards steps will be more frequent than upwards steps and the particle will on average fall down. The position of the particle is measured with the help of a CCD camera. Each time the particle is observed to jump upwards, this information is used to insert a potential barrier that hinders the particle to move down. By repeating this procedure, the average particle motion is now upstairs and work is done against the potential gradient. By lifting the particle mechanical work has therefore been produced by gathering information about its position. This is the first example of a device that converts information into energy for a system coupled to a single thermal environment. However there is not a contradiction with the second law because Sagawa and Ueda formalized the idea that information gained through microlevel measurements can be used to extract added work from a heat engine. [21] Their formula for the the maximum extractable work is:

$$\langle W_{max} \rangle = -\Delta F + k_B\ T\ \langle I \rangle, \tag{1}$$

where ΔF is the free energy difference between the final and initial state and the extra term represents the so-called mutual information I. In absence of measurement errors this quantity reduces to the Shannon entropy : $I = -\sum_k P(\Gamma_k) \ln[P(\Gamma_k)]$, where $P(\Gamma_m)$ is the probability of finding the system in the state Γ_k. Then in the

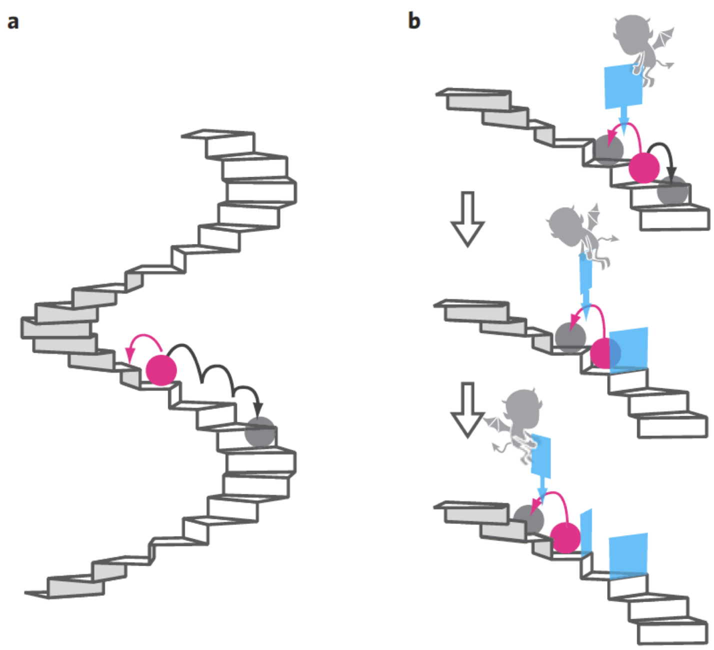

Figure 4: (a) Experimental realization of Szilard's engine. (a) A colloidal particle in a staircase potential moves downwards on average, but energy fluctuations can push it upwards from time to time. (b) When the demon observes such an event, he inserts a wall to prevent downward steps. By repeating this procedure, the particle can be brought to move upwards, performing work against the force created by the staircase potential. In the actual experiment, the staircase potential is implemented by a tilted periodic potential and the insertion of the wall is simply realized by switching the potential, replacing a minimum (no wall) by a maximum (wall) (adapted from Ref. [20]).

specific case of the previously described staircase potential [20]: $I = -p \ln p - (1 - p) \ln p$ where p is the probability of finding the particle in a specific region.

In this context the Jarzynski equality (see appendix A) also contains this extra term and it becomes :

$$\langle \exp(-\beta W + I) \rangle = \exp(-\beta \Delta F), \tag{2}$$

which leads to

$$\langle W \rangle \geq \Delta F - k_B T \langle I \rangle. \tag{3}$$

Equation (2) and (3) generalize the second law of thermodynamics taking into account the amount of information introduced into the system [22, 9]. Indeed Eq. (3) indicates that thank to information the work performed on the system to drive it between an initial and a final equilibrium states can be smaller than the free energy difference between the two states. Equation (2) has been directly tested in a single electron transistor [23].

2.1.2 *The autonomous Maxwell demon improves cooling*

An autonomous Maxwell demon using a local feedback mechanism which allows an efficient cooling of the system [24, 25]. The device, whose principle is sketched in

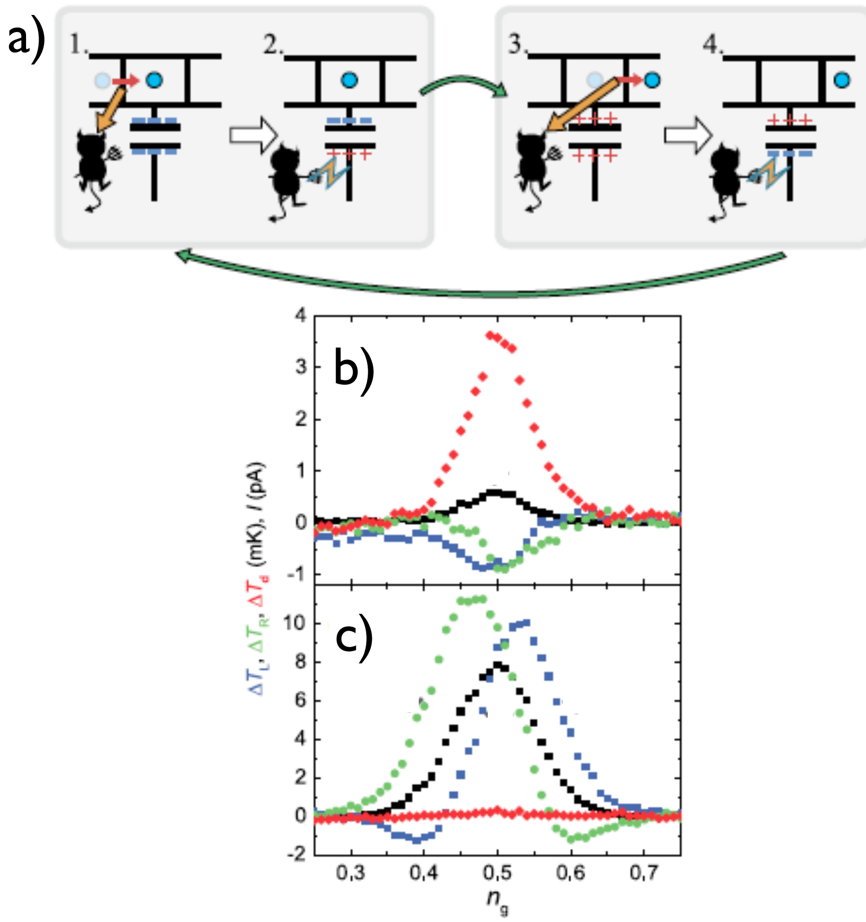

Figure 5: **a)** Principle of the experimental realization of the autonomous Maxwell demon. The horizontal top row schematizes a Single Electron Transistor. Electrons (blue circle) can tunnel inside the central island from the left wall and outside from the right wall. The demon watches at the state of the island and it applies a positive charge to attract the electrons when they tunnel inside and they repels them when they tunnel outside. The systems cools because of the energy released toward the heat bath by the tunneling events and the presence of the demon makes the cooling processes more efficient. The energy variation of the processes is negative because of the information introduced by the demon. **b)** The measured temperature variations of the left (bleu line) and right (green line) leads as a function of the external control parameter n_g when the demon is active and the bath temperature is $50mK$. We see that at the optimum value $n_g = 1/2$ both leads are cooled of about $1mK$ and the current I_e flowing through the SET (black line) has a maximum. At the same time in order to processes information the temperature of the demon (red line) increases of a few mK. **c)** The same parameter of the panel b) are measured when the demon is not active. We see that the demon temperature does not change, whereas both leads are now heated by the current I_e (adapted from Ref. [24]).

Fig. 5a), is composed by a SET (Single Electron Transitor) formed by a small normal metallic island connected to two normal metallic leads by tunnel junctions, which permit electron transport between the leads and the island. The SET is biased by a potential V and a gate voltage V_g, applied to the island via a capacitance, controls the current I_e flowing through the SET. The island is coupled capacitively with a single electron box which acts as a demon which detects the presence of an electron in the island and applies a feedback. Specifically when an electron tunnels to the island, the demon traps it with a positive charge (panels 1 and 2). Conversely, when an electron leaves the island, the demon applies a negative charge to repel further electrons that would enter the island (panels 3 and 4). This effect is obtained by designing the electrodes of the demon in such a way that when an electron enters the island from a source electrode, an electron tunnels out of the demon island as a response, exploiting the mutual Coulomb repulsion between the two electrons. Similarly, when an electron enters to the drain electrode from the system island, an electron tunnels back to the demon island, attracted by the overall positive charge. The cycle of these interactions between the two devices realizes the autonomous demon, which allows the cooling of the leads. In the experimental realization presented in [24], the leads and the demon were thermally insulated, and the measurements of their temperatures is used to characterize the effect of the demon on the device operation. In Fig. 5b) we plot the variation of the leads temperatures as a function of $n_g \propto V_g$ when the demon acts on the system. We clearly see that around $n_g = 1/2$ the two leads are both cooled of $1mK$ at a mean temperature of $50mK$. This occurs because the tunneling electrons have to take the energy from the thermal energy of the leads, which, being thermally isolated, cool down. This increases the rate at which electrons tunnel against Coulomb repulsion, giving rise to increased cooling power. At the same time the demon increases its temperature because it has to dissipate energy in order to processes information, as discussed in Ref. [26]. Thus the total (system+demon) energy production is positive. The coupling of the demon with the SET can be controlled by a second gate which acts on the single electron box. In Fig. 5c) we plot the measured temperatures when the demon has been switched off. We clearly see that in such a case the demon temperature does not change and the two electrodes are heating up because of the current flow. This is the only example which shows that under specific conditions an autonomous local Maxwell demon, which does not use the external feedback, can be realized.

2.2 Experiments on Landauer's principle

The experiments in the last section show that one can extract work from information. In the rest of this section we will discuss the reverse process, i.e. the energy needed to erase information. By applying the second law of thermodynamics, Landauer demonstrated that information erasure is necessarily a dissipative process: the erasure of one bit of information is accompanied by the production of at least $k_B T \ln(2)$ of heat into the environment (see Box1). This result is known as Landauer's erasure principle. It emphasizes the fundamental difference between the process of writing and erasing information. Writing is akin to copying information from one device to another: state left is mapped to left and state right is mapped to right, for example. This one-to-one mapping can be realized in principle without dissipating any heat (in statistical mechanics one would say that it conserves the volume in phase space). By contrast, erasing information is a two-to-one transformation: states left and right are mapped onto one single state, say right (this process does not conserve the volume in phase space and is thus dissipative).

Landauer's original thought experiment has been realized for the first time in a real system in 2011 using a colloidal Brownian particle in a fluid trapped in a double-well potential produced by two strongly focused laser beams [3, 4, 5] (see also Appendix

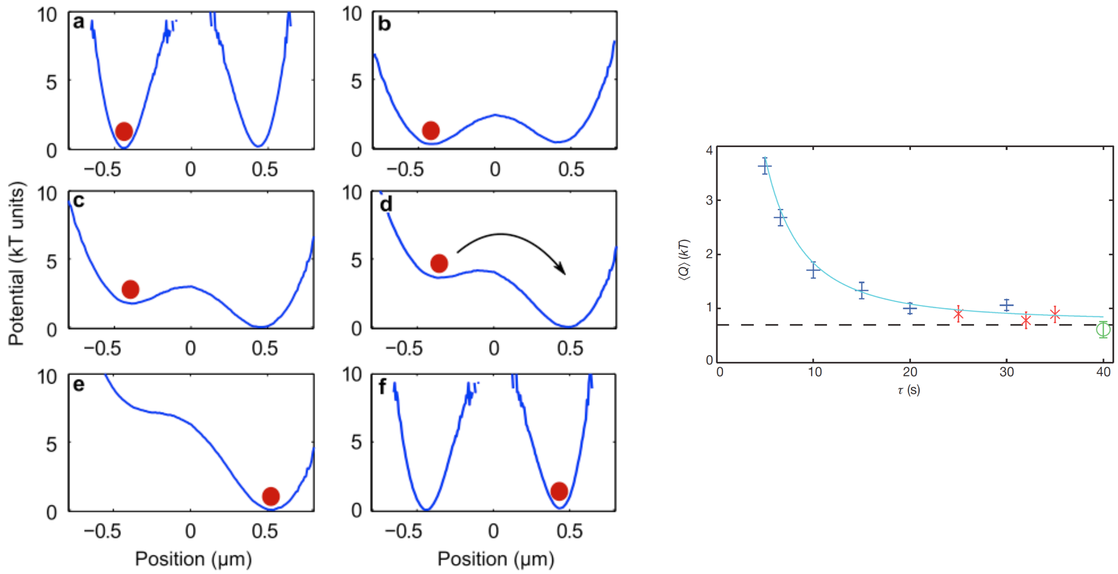

Figure 6: Experimental verification of Landauer's erasure principle. A colloidal particle is initially confined in one of two wells of a double-well potential with probability one-half. This configuration stores one bit of information. By modulating the height of the barrier and applying a tilt, the particle can be brought to one of the wells with probability one, irrespective of the initial position. This final configuration corresponds to zero bit of information. In the limit of long erasure cycles, the heat dissipated during the erasure process can approach, but not exceed, the Landauer bound indicated by the dashed line (adapted from Ref. [3] for details).

B). This system has two distinct states (particle in the right or left well) and may thus be used to store one bit of information. The erasure principle has been verified by implementing a protocol proposed by Bennett and illustrated in Fig. 6. At the beginning of the erasure process, the colloidal particle may be either in the left or right well with equal probability of one half. The erasure protocol is composed of the following steps: 1) the barrier height is first decreased by varying the laser intensity, 2) the particle is then pushed to the right by gently inclining the potential and 3) the potential is brought back to its initial shape. At the end of the process, the particle is in the right well with unit probability, irrespective of its departure position. As in the previous experiment, the position of the particle is recorded with the help of a camera. For a full erasure cycle, the average heat dissipated into the environment is equal to the average work needed to modulate the form of the double-well potential. This quantity was evaluated from the measured trajectory and shown to be always larger than the Landauer bound which is asymptotically approaches in the limit of long erasure times. However, in order to reach the bound, the protocol must be accurately chosen because as discussed in Ref. [3] and shown experimentally [27] there are protocols that are intrinsically irreversible no matter how slow are performed. The way in which a protocol can be optimized has been theoretically solved in Ref. [28] but the optimal protocol is not often easy to apply in an experiment

2.3 Other experiments on the physics of information

By having successfully turned gedanken into real experiments, the above four seminal examples provide a firm empirical foundation to the physics of information and the intimate connection existing between information and energy. This connection is reinforced by the relationship between the generalized Jarzinsky equality [29] and the Landauer bound which has been proved and tested on experimental data in Ref. [4] and shortly summarized in the appendix A of this chapter .

A number of additional experiments have verified the erasure principle in various systems [30, 31, 32, 33, 34, 35, 36]. The latter include an electrical RC circuit [30] and a feedback trap [32, 33]. In addition, Ref. [34] has studied the symmetry breaking, induced in the probability distribution of the position of a Brownian particle, by commuting the trapping potential from a single to a double well potential. The authors measured the time evolution of the system entropy and showed how to produce work from information. Finally, experiments on the Landauer bound have been performed in nano devices, most notably using a single electron box [31] and nanomagnets [35, 36]. These experiments open the way to insightful applications for future developments of information technology.

3 Extensions to the quantum regime

3.1 Experiments on quantum Maxwell's demon

The experimental investigation of the physics of information has lately been extended to the quantum regime. The group of Roberto Serra in Sao Paulo has successfully realized a quantum Maxwell demon in a Nuclear Magnetic Resonance (NMR) setup [37]. The demon was implemented as a spin-1/2 quantum memory that acquires information about another spin-1/2 system and employs it to control its dynamics. Using a coherent measured-based feedback protocol, the demon was shown to rectify the nonequilibrium entropy production due to quantum fluctuations and produce useful work. Concretely, the demon gained information about the system via a complete projective measurement. Based on the outcome of this measurement, a controlled evolution was applied to the system to balance the entropy production. Using quantum state tomography to reconstruct the density matrix ρ of the system at all times, the produced average work $\langle W \rangle$, or equivalently the mean entropy production $\langle \Sigma \rangle = \beta(\langle W \rangle - \Delta F)$, was shown to be bounded by the information gain, $\langle \Sigma \rangle \leq I_{\text{gain}}$. The latter quantifies the average information that the demon obtains by reading the outcomes of the measurement and is defined as $I_{\text{gain}} = S(\rho) - \sum_i p_i S(\rho_i)$, where ρ_i is the state after a measurement which occurs with probability p_i (see Fig. 7).

More recently, a quantum Maxwell demon has been implemented in a circuit QED system [38]. Here, the demon was a microwave cavity that encodes quantum information about a superconducting qubit and converts that information into work by powering up a propagating microwave pulse by stimulated emission. The power extracted from the system was directly accessed by measuring the difference between incoming and outcoming photons of the cavity. Using full tomography of the system, the entropy remaining in the demon's memory was further quantified and was shown to be always higher that the system entropy decrease, in agreement with the second law.

In addition in a quantum demon setting a multi-photon optical interferometer allowed the measure of the extractable work which was used as a thermodynamic

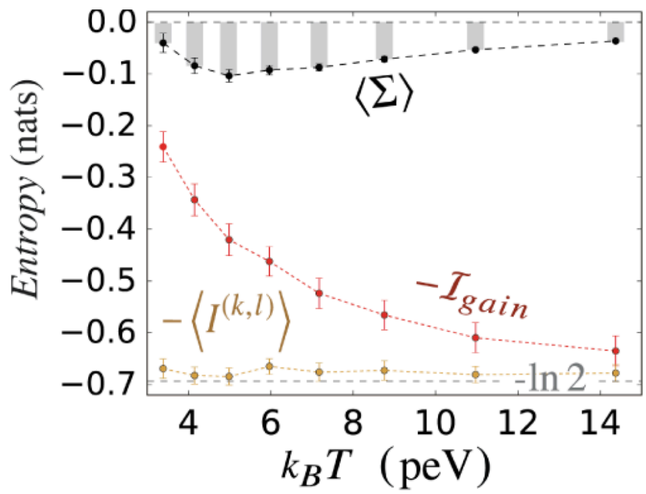

Figure 7: Thermodynamics of a quantum Maxwell demon. Verification of the second law for the nonequilibrium mean entropy production, $\langle \Sigma \rangle = \beta(\langle W \rangle - \Delta F) \leq I_{\text{gain}}$, in the presence of quantum feedback as a function of temperature. The parameter I_{gain} quantifies the information gained through the measurement (adapted from Ref. [37]).

separability criterion to assess the entanglement of two-qubit and three-qubit systems [39]. An experimental analysis of two-qubit Bell states and three-qubit GHZ and W states has confirmed that more work can be extracted from an entangled state than from a separable state. Bounds on the extractable work can therefore be employed as a useful thermodynamic entanglement witness.

3.2 Experiments on quantum Landauer's principle

Erasure of information encoded in quantum states has been first theoretically considered by Lubkin [41] and Vedral [42] (see also Ref. [12]). An experimental verification of the Landauer principle in a quantum setting has been recently reported using a molecular nanomagnet at a temperature of 1K [40]. One bit of information was initially stored in a double-well potential of collective giant spin $S_z = \pm 10$ of a Fe_8 molecule. Work for the application of the tilt induced by a transverse magnetic field was determined via measurements of the magnetic susceptibility. Contrary to classical erasure which is achieved by decreasing the barrier height, here erasure was promoted by a thermally activated quantum tunnelling process. As a result, full erasure can be achieved much faster than in the classical regime. Using the product of the erasure work and the relaxation time, $W \cdot \tau_{\text{rel}}$, as a figure of merit for the energy-time cost of information erasure, this experiment has reached the lower value to date with $W \cdot \tau_{\text{rel}} \simeq 2 \cdot 10^{-23}$ erg/bit. As compared to 10^{-12} erg/bit. s for the classical experiment with the colloidal particle [3]. This puts the experiment close to the fundamental limit imposed by the Heisenberg uncertainty relation (see Fig. 8).

4 Applications

Landauer's principle applies not only to information erasure but also to all logically irreversible devices that possess more outputs than inputs. Thus, any Boolean

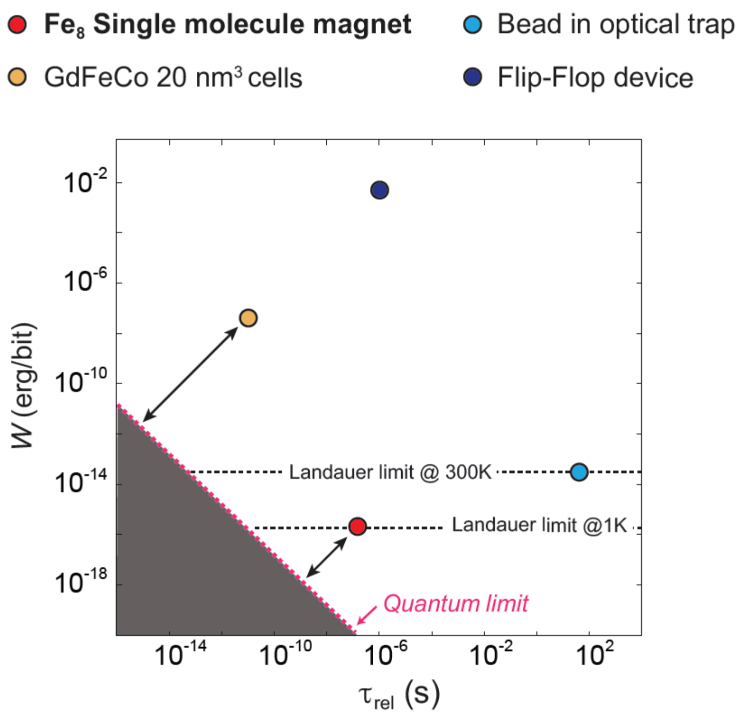

Figure 8: Energy-time cost of erasure. The diagram shows the product of the energy and the time needed for erasure, $W \cdot \tau_{\text{rel}}$, for various systems. The quantum limit is given by the Heisenberg uncertainty relation, $E \cdot \Delta t \geq \pi\hbar/2$. The Fe_8 molecule is currently the closest to the quantum limit (red dot) (adapted from Ref. [40]).

gate operation that maps several input states onto the same output state, such as AND, NAND, and OR, have several states which are logically irreversible and will lead to the dissipation of an amount of heat of $k_B T \ln(2)$ per processed bit, akin to the erasure process. As a result, Landauer's principle has important technological consequences. Heating laptops are nowadays becoming part of everyday experience. Heat production in microprocessors used in modern computers is known to be a major factor hindering their miniaturization, as it gets more and more difficult to evacuate excess heat when size, and thus surface, is reduced. While the overall heat dissipated in microchips is steadily decreasing, it still several orders of magnitude larger than the Landauer limit. However, the switching energy of a CMOS/FET transistor is predicted to reach the Landauer bound by 2035, indicating that engineers will soon face a fundamental physical limitation imposed by the second law of thermodynamics [43, 44]. This is remarkable as $k_B T \ln(2)$ is about 3.10^{-21} Joule at room temperature and hence 22 orders of magnitude smaller than typical energy dissipated on our macroscopic scale. Recently, an experiment has demonstrated that Maxwell's demon can generate electric current and power by rectifying individual randomly moving electrons in small transistors [45].

Man-made computers are not the only existing information processing devices. Scientists have long realized that living biological cells can be viewed as biochemical information processors that may even outperform our current technology [46]. Cells are, for example, able to reproduce and create copies of themselves, acquire and process information coming from external stimuli, as well as communicate and exchange information with other cells. Recently, Landauer's principle has been employed to evaluate the energetic cost of a living cell computing the steady-state concentration of a chemical ligand in its surrounding environment [47]; it has been argued that it sets strong constraints on the design of cellular computing networks, as there is a tradeoff between the information processing capability of such a network and its energetic cost. Another important problem is the investigation of ultrasensitive switches in molecular biology. A concrete example is the flagellar motor of E. coli bacteria that switches from clockwise to counterclockwise rotation depending on the intracellular concentration of a regulator protein. Switching mechanisms are highly complex and not fully understood. A mathematical framework that models the sensing of the protein concentration by the flagellar motor as a Maxwell demon has been successfully developed to calculate the rate of energy consumption needed to both sense and switch, and provide a quantitative description of the switching statistics [48]. More recent work has focused on the efficiency of cellular information processing [49], biochemical signal transduction [50], as well as on cost and precision of Brownian clocks [51] and computational copying in biochemical systems [52].

Maxwell's demon is therefore still vibrant 150 years after its inception. Together with Landauer's principle, he continues to play a prominent role in modern research as illustrated by the last examples. Having only very recently become an experimental science, information physics appears to have a promising future ahead.

APPENDICES

A Stochastic thermodynamics and information energy cost

When the size of a system is reduced the role of fluctuations (either quantum or thermal) increases. Thus thermodynamic quantities such as internal energy, work, heat and entropy cannot be characterized only by their mean values but also their fluctuations and probability distributions become relevant and useful to make predictions on a small system. Let us consider a simple example such as the motion of a Brownian particle subjected to a constant external force. Because of thermal fluctuations, the work performed on the particle by this force per unit time, i.e., the injected power, fluctuates and the smaller the force, the larger is the importance of power fluctuations [53, 54, 55]. The goal of stochastic thermodynamics is just that of studying the statistical properties of the above mentioned fluctuating thermodynamic quantities in systems driven out of equilibrium by external forces, temperature differences and chemical reactions. For this reason it has received in the last twenty years an increasing interest for its applications in microscopic devices, biological systems and for its connections with information theory[53, 54, 55].

Specifically it can be shown that the fluctuations on a time scale τ of the internal energy ΔU_τ, the work W_τ and the heat Q_τ are related by a first principle like equation, i.e.,

$$\Delta U_\tau = U(t+\tau) - U(t) = \tilde{W}_\tau - Q_\tau \tag{4}$$

at any time t.

Furthermore the statistical properties of energy and entropy fluctuations are constrained by fluctuations theorems which impose bounds on their probability distributions (for more details see ref.[53, 54, 55]). We summarize in the next section one of them which is can be related to information and to Landauer's bound.

A.1 Estimate the free energy difference from work fluctuations

In 1997 [56, 57] Jarzynski derived an equality which relates the free energy difference of a system in contact with a heat reservoir to the pdf of the work performed on the system to drive it from A to B along any path γ in the system parameter space. Specifically, when a system parameter λ is varied from time $t=0$ to $t=t_s$, Jarzynski defines for one realization of the "switching process" from A to B the work performed on the system as

$$W_{\mathrm{st}} = \int_0^{t_s} \dot{\lambda}\, \frac{\partial H_\lambda[z(t)]}{\partial \lambda} \mathrm{d}t, \tag{5}$$

where z denotes the phase-space point of the system and H_λ its λ-parametrized Hamiltonian [1]. One can consider an ensemble of realizations of this "switching process" with initial conditions all starting in the same initial equilibrium state. Then W_{st} may be computed for each trajectory in the ensemble. The Jarzynski equality states that [56, 57]

$$\exp\left(-\beta \Delta F\right) = \langle \exp\left(-\beta W_{\mathrm{st}}\right)\rangle, \tag{6}$$

where $\langle\cdot\rangle$ denotes the ensemble average, $\beta^{-1} = k_B T$ with k_B the Boltzmann constant and T the temperature. In other words $\langle \exp\left[-\beta W_{\mathrm{diss}}\right]\rangle = 1$, since we can always write $W_{\mathrm{st}} = \Delta F + W_{\mathrm{diss}}$ where W_{diss} is the dissipated work. Thus it is easy to see that there must exist some paths γ such that $W_{\mathrm{diss}} \leq 0$. Moreover, the inequality $\langle \exp x\rangle \geq \exp\langle x\rangle$ allows us to recover the second principle, namely $\langle W_{\mathrm{diss}}\rangle \geq 0$, i.e. $\langle W_{\mathrm{st}}\rangle \geq \Delta F$.

A.2 Landauer bound and the Jarzynski equality

We discuss in this appendix the strong relationship between the Jarzynski equality and the Landauer's bound. In Box 1 we presented the Landauer's principle as related to the system entropy. Let us consider as a specific example the experiment on the colloidal particle described in section 2.2 [4]. In the memory erasure procedure which forces the system in the state 0, the entropy difference between the final and initial state is $\Delta S = -k_{\mathrm{B}} \ln(2)$. In contrast the internal internal energy is unchanged by the protocol. Thus it is natural to await $\Delta F = k_{\mathrm{B}} T \ln(2)$. However the ΔF that appears in the Jarzynski equality is the difference between the free energy of the system in the initial state (which is at equilibrium) and the equilibrium state corresponding to the final value of the control parameter: $F(\lambda(\tau)) - F(\lambda(0))$. Since the height of the barrier is always finite there is no change in the equilibrium free energy of the system between the beginning and the end of the procedure. Then $\Delta F = 0$, which implies $\left\langle e^{-\beta W_{\mathrm{st}}} \right\rangle = 1$. Thus it seems that there is a problem between the Landauer principle (see Box 1) and the Jarzynski equality of Eq. 6.

Nevertheless Vaikuntanathan and Jarzynski [29] have shown that when there is a difference between the actual state of the system (described by the phase-space

[1] This is a more general definition of work and it coincides with the standard one only if λ is a displacement (for more details see ref.[55]

density ρ_t), and the equilibrium state (described by ρ_t^{eq}), the Jarzynski equality can be modified:

$$\left\langle e^{-\beta W_{\mathrm{st}}(t)} \right\rangle_{(x,t)} = \frac{\rho^{\mathrm{eq}}(x,\lambda(t))}{\rho(x,t)} e^{-\beta \Delta F(t)}, \tag{7}$$

where $\langle . \rangle_{(x,t)}$ is the mean on all the trajectories that pass through x at time t.

In the experiment presented in section 2.2, the selection of the trajectories where the information is actually erased, corresponds to fix x to the chosen final well at the time $t = \tau$. It follows that $\rho(0, \tau)$ is the probability of finding the particle in the targeted state 0 at the time τ. Indeed because of the very low energy measured in the protocol thermal fluctuations play a role and the particle can be found in the wrong well at time τ, i.e. the proportion of success P_S of the procedure is equal to $\rho(0, \tau)$. In contrat the equilibrium distribution is $\rho^{\mathrm{eq}}(0, \lambda(\tau)) = 1/2$. Then:

$$\left\langle e^{-\beta W_{\mathrm{st}}(\tau)} \right\rangle_{\to 0} = \frac{1/2}{P_S}. \tag{8}$$

Similarly for the trajectories that end the procedure in the wrong well (i.e. state 1) we have:

$$\left\langle e^{-\beta W_{\mathrm{st}}(\tau)} \right\rangle_{\to 1} = \frac{1/2}{1 - P_S}. \tag{9}$$

Taking into account the Jensen's inequality, i.e. $\langle e^{-x} \rangle \geq e^{-\langle x \rangle}$, we find that equations 8 and 9 imply:

$$\begin{array}{l} \langle W_{\mathrm{st}} \rangle_{\to 0} \geq k_{\mathrm{B}} T \left[\ln(2) + \ln(P_S) \right] \\ \langle W_{\mathrm{st}} \rangle_{\to 1} \geq k_{\mathrm{B}} T \left[\ln(2) + \ln(1 - P_S) \right] \end{array}. \tag{10}$$

Notice that the mean work dissipated to realize the procedure is simply:

$$\langle W_{\mathrm{st}} \rangle = P_S \times \langle W_{\mathrm{st}} \rangle_{\to 0} + (1 - P_S) \times \langle W_{\mathrm{st}} \rangle_{\to 1}, \tag{11}$$

where $\langle . \rangle$ is the mean on all trajectories. Then using the previous inequalities it follows:

$$\langle W_{\mathrm{st}} \rangle \geq k_{\mathrm{B}} T \left[\ln(2) + P_S \ln(P_S) + (1 - P_S) \ln(1 - P_S) \right], \tag{12}$$

which is indeed the generalization of the Landauer's bound for $P_S < 1$. In the limit case where $P_S \to 1$, we have:

$$\left\langle e^{-\beta W_{\mathrm{st}}} \right\rangle_{\to 0} = 1/2. \tag{13}$$

Since this result remains approximatively verified for proportions of success close enough to 100%, it explains why in the experiment we find $\Delta F_{\mathrm{eff}} \approx k_{\mathrm{B}} T \ln(2)$.

This result is useful because it strongly binds the generalized Jarzynski equality (a thermodynamic relation) to Landauer's bound.

A.2.1 Experimental test of the generalized Jarzynski equality

The theoretical predictions of the previous section have been checked in the experiment on the colloidal particle presented in Section 2.2 and in Fig.6. In ref. [4] the authors experimentally compute ΔF_{eff} which is the logarithm of the exponential average of the dissipated heat for trajectories ending in state 0:

$$\Delta F_{\mathrm{eff}} = -\ln\left(\left\langle \mathrm{e}^{-\beta W_{\mathrm{st}}}, \right\rangle_{\to 0} \right) \tag{14}$$

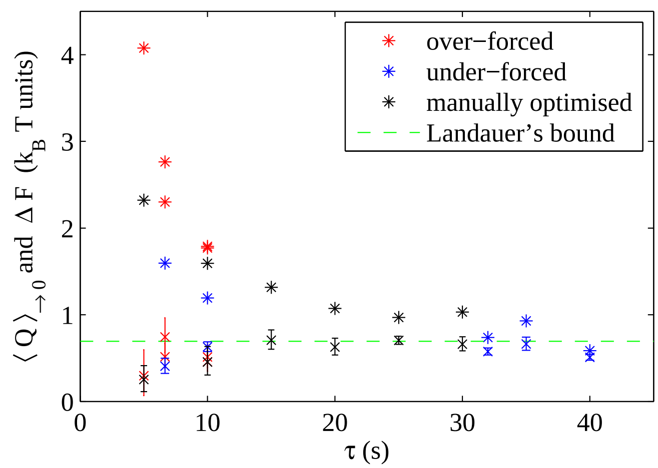

Figure 9: Mean dissipated heat ($*$) and effective free energy difference (Eq. 14) ($\times$) for several procedures, with fixed τ and different values of $F_{\max}$. The red points have a force too high, and a $P_S \geq 99\,\%$. The blue points have a force too low and $91\,\% \leq P_S < 95\,\%$ (except the last point which has $P_S \approx 80\,\%$). The black points are considered to be optimised and have $95\,\% \leq P_S < 99\,\%$(adapted from Ref[5]).

where $W_{\rm st}$ in the experiment is equal to Q as explained in Appendix B.2 (Eq. 17).

Data are shown in Fig. 9. The error bars are estimated by computing the average on the data set with 10% of the points randomly excluded, and taking the maximal difference in the values observed by repeating this operation 1000 times. Except for the first points ($\tau = 5\,$s), the values of $\Delta F_{\rm eff}$ are very close to $k_{\rm B}T\ln 2$ for any value of the protocol duration τ, which is in agreement with equation 9, since P_S is close to 100 %. Hence, we retrieve the Landauer's bound for the free-energy difference, for any duration of the information erasure procedure.

Note that this result is not in contradiction with the classical Jarzynski equality, because if we average over all the trajectories (and not only the ones where the information is erased), we should find $\left\langle {\rm e}^{-\beta W_{\rm st}} \right\rangle = 1$.

In ref.[4, 5] the authors looked at the two sub-procedures $1 \to 0$ and $0 \to 0$ separately, finding an excellent agreement with Eq. 12.

B Set-up used in the experiment presented in Section 2.2

B.1 The one-bit memory system

The one-bit memory system, is made of a double well potential where one particle is trapped by optical tweezers. If the particle is in the left-well the system is in the state "0", if the particle is in the right-well the system in the state "1". Full details of the experimental set-up can be found in ref.[5] and we summarize here the main features. We use a custom-built vertical optical tweezer made of an oil-immersion objective (63×, N.A. = 1.4) which focuses a laser beam (wavelength λ = 1064nm) to the diffraction limit for trapping glass beads (2μm in diameter) [5]. The beads are dispersed in bidistilled water in very small concentration. The suspension is introduced in a disk-shaped cell (18mm in diameter, 1mm in depth), a single bead is then trapped and moved away from the others. This step is quite important to avoid that during the measurement the trapped bead is perturbed by other Brownian particles. The position of the bead is tracked using a fast camera with a resolution of 108nm/pixels, which gives after treatment the position with an accuracy better than 10nm. The trajectories of the bead are sampled at 502Hz. The double-well potential is obtained by switching the laser at a rate of 10KHz between two points at a distance $d_f = 1.45\mu$m, kept fixed. The form of the potential, which is a function of d_f and the laser intensity I_L, can be determined in equilibrium by measuring the probability

$$P(x, I_L) = A \exp[-U_o(x, I_L)/(kT)] \tag{15}$$

of the position x of the bead, i.e. $U_o(x, I_L) = -kT \ln[P(x, I_L)/A]$ (see Fig. 6 a,b and f) The distribution $P(x, I_L)$ is estimated on about 10^6 samples. The measured $U_o(x, I_L)$ are plotted in Figs. 6 a,b and f) and can be fitted by an eighth order polynomial $U_o(x, I_L) = \sum_{n=0}^{8} u_n(I_L, d_f)x^n$. The distance between the two minima of the double-well potential is 0.9μm. The two wells are nearly symmetric with a maximum energy difference of $0.4kT$. The height of the barrier is modulated by varying the power of the laser from $I_L = 48mW$ (barrier height $> 8kT$) to $I_L = 15mW$ (barrier height $= 2.2kT$). In equilibrium for a barrier of $8kT$, the characteristic jumping time (Kramers time) between the two wells is about 3000s which is much longer than any experimental time.

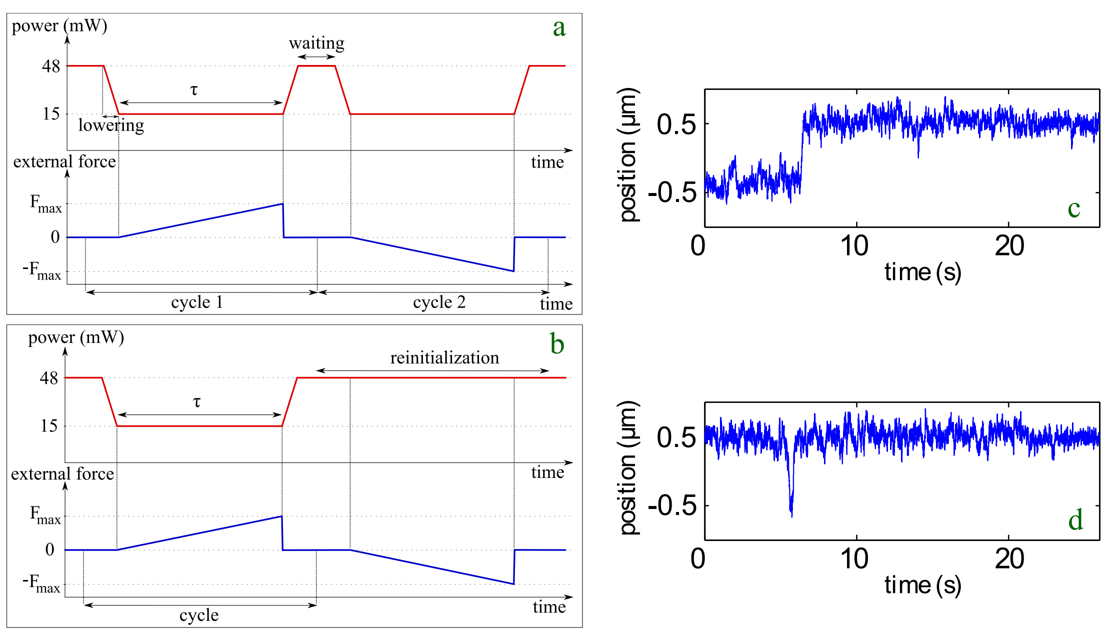

Figure 10: Erasure cycles and typical trajectories. a) Protocol used for the erasure cycles bringing the bead from left (state 0) to right (state 1), and vice versa. b) Protocol used to measure the heat for the cycles for which the bead does not change wells. The reinitialization is needed to restart the measurement, but is not a part of the erasure protocol. c) Example of a measured bead trajectory for the transition $0 \to 1$. d) Example of a bead trajectory for the transition $1 \to 1$.

The external tilt is created by displacing the cell with respect to the laser with a piezo, thus inducing a viscous flow. The viscous force is simply $F = -\gamma v$, where $\gamma = 1.89 N.s.m^{-1}$ is the friction coefficient and v the velocity of the cell. In the erasure protocol, the amplitude of the viscous force is increased linearly during time τ, $F(t) = F_{max}\, t/\tau$. In Figs. 6 a,b and f) , we plot $U(x,t) = U_o(x, I_L) - F(t)\, x$ for $I_L = 15mW$ and for three different values of t. The typical erasure protocol is presented in Fig.10. The reinitialization procedure shown in this figure is necessary to displace the cell to its initial position but it does not contribute to the erasure process. Notice that, contrary to the useful erasure cycles, this reinitialization is performed when the barrier is high. Thus the bead remains always in the same well. Note that for the theoretical procedure, the system must be prepared in an equilibrium state with same probability to be in state 1 than in state 0. However, it is more convenient experimentally to have a procedure always starting in the same position. Therefore we separate the procedure in two sub-procedures: one where the bead starts in state 1 and is erased in state 0, and one where the bead starts in state 0 and is erased in state 0. The fact that the position of the bead at the beginning of each procedure is actually known is not a problem because this knowledge is not used by the erasure procedure. The important points are that there is as many procedures starting in state 0 than in state 1, and that the procedure is always the same regardless of the initial position of the bead. Examples of trajectories for the two sub-procedures $1 \to 0$ and $0 \to 0$ are shown in Fig.10.

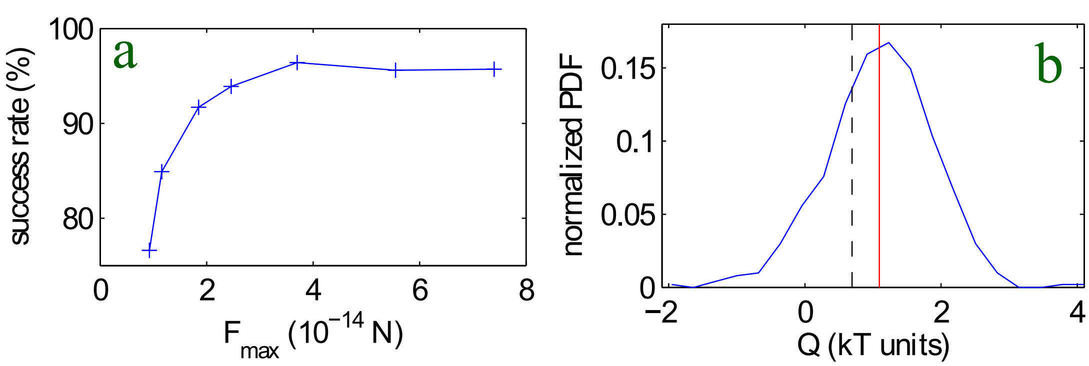

Figure 11: a) Success rate P_S of the erasure cycle as a function of the maximum tilt amplitude F_{max}. b) Heat distribution $P(Q)$ for transition $0 \to 1$ with $\tau = 25s$ and $F_{max} = 1.89 \times 10^{-14}$N. The solid vertical line indicates the mean dissipated heat, and the dashed vertical line marks the Landauer bound.

A key characteristic of the erasure process is its success rate P_S, that is, the relative number of cycles bringing the bead in the expected well. Fig.11a) shows the dependence of the erasure rate on the tilt amplitude, F_{max}. For definiteness, we have kept the product $F_{max}\tau$ constant. We observe that the erasure rate drops sharply at low amplitudes when the tilt force is too weak to push the bead over the barrier, as expected. For large values of F_{max}, the erasure rate saturates at around 95%. This saturation reflects the finite size of the barrier and the possible occurrence of spontaneous thermal activation into the wrong well. An example of a distribution of the dissipated heat for the transition $0 \to 1$ is displayed in Fig.11b). Owing to thermal fluctuations, the dissipated heat may be negative and maximum erasure below the Landauer limit may be achieved for individual realizations, but not on average [3, 5].

B.2 Heat measurements

The heat dissipated by the tilt is

$$Q = \int_{T_{low}}^{T_{low}+\tau} dt\, F(t)\dot{x}(t) = \int_{T_{low}}^{T_{low}+\tau} dt\, F_{max}\,\frac{t}{\tau}\dot{x}(t), \quad (16)$$

where T_{low} is the time at which the barrier is reduced to its minimum value. The velocity is computed using the discretization, $\dot{x}(t+\Delta t/2) \simeq [x(t+\Delta t) - x(t)]/\Delta t$. To characterize the approach to the Landauer limit we note that, in the quasi-static limit, the mean work $< W >$ can be expressed in terms of the free energy difference ΔF as $< W > \simeq \Delta F + C/\tau$ [53](see Appendix A). According to the first law of thermodynamics, $< \Delta U > = < W > - < Q > = 0$ for a cycle. As a result, $\Delta F = -T\Delta S$ and $< Q > = < W > \simeq kT\ln 2 + C/\tau$.

Finally we notice that in the used protocol the $W_{\rm st}$ defined in Eq. 5 is equal to Q. Indeed as $F(t = T_{\rm low}) = 0 = F(t = T_{\rm low} + \tau)$ it follows from an integration by parts that the stochastic work is equal to the heat dissipated by the force:

$$W_{\rm st} = \int_{T_{\rm low}}^{T_{\rm low}+\tau} -\dot{F}x\,{\rm d}t' = \int_{T_{\rm low}}^{T_{\rm low}+\tau} F\dot{x}\,{\rm d}t' = Q. \quad (17)$$

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